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TESIS DOCTORAL

ESSAYS ON FIRM DYNAMICS

Autor:

Román Fossati

Director:

Susanna Esteban and Nezih Guner

DEPARTAMENTO DE ECONOMÍA

Getafe, 31de Octubre de 2013

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Firma del Tribunal Calificador:

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Para Virginia y Lara,

y para mi familia en Argentina.

Acknowledgements

I would like to thank to my advisors, Susanna Esteban and Nezih Guner, for their continuous guidance and support, for so many fruitful and stimulating discussions, and for being generous with their time. They were helpful and influential at the time to make important decisions. I was extremely fortunate to have them as my advisors. What I have learned from them goes far beyond what is reflected in this dissertation.

Working with Andrés Erosa in a joint paper was one of the most wonderful experiences of my Ph.D. I want to express my gratitude to Andrés Erosa for his constant motivation and optimism and for guiding me with the generosity and enthusiasm that characterizes his huge human quality.

I also appreciate valuable comments from other professors. Antonia Diaz, Matthias Kredler and Josep Pijoan-Mas gave me very important suggestions to different chapters of this dissertation. Antonio Cabrales, Marco Celentani, Pedro Gomes, Belen Jerez, Matilde Machado, Carlos Ponce, Victor Rios-Rull, Manuel Santos, Ludo Visschers and Ulrich Wagner also gave me helpful advices.

I would like to specially thank my friend Heiko Rachinger for his support during all these years. He listened to my presentations many times and gave me many comments to improve my research. I was very lucky to have crossed paths with him as well as with Ming-Jin Jiang. I spent so much time with both of them studying together and sharing some mates. I really appreciate all that.

I am also grateful to all the graduate students, Efi, Daniel, Dolo, Georgi, Joaquin, Juan, Lian, Lucila, Luifa, and Luis that gave me valuable comments on my research, for studying together, and for sharing so many good times.

Most importantly, I want to thank my wife Virginia for her support during all these years and for always being there for me, as well as to my family, my parents Lipan and Juan Carlos, my brothers Matias y German, for their unconditional support and patience. To them I dedicate this thesis. Thank you for everything!!!

Resumen en Castellano

En mi tesis doctoral, se desarrollan modelos de dinámica de firmas para estudiar los determinantes de la integración vertical dentro y entre industrias, el efecto de políticas que dependen del tamaño de las firmas, y el impacto de la regulación dual en el mercado de trabajo sobre la productividad agregada y la distribución de firmas.

En el primer capítulo, **“Outsourcing versus Vertical Integration: A Dynamic Model of Industry Equilibrium”**, se desarrolla un modelo estocástico de una industria con empresas heterogéneas que interactúan como compradoras y vendedoras de insumos en un contexto con fricciones de mercado las cuales inducen un problema de "hold-up" a los productores de bienes finales. El modelo genera un comportamiento consistente con nuevos hechos empíricos sobre integración vertical los cuales indican que las empresas verticalmente integradas son más productivas, más grandes y están formadas por las mejores empresas productoras de insumos (en términos de tamaño y productividad). En este contexto, la estructura vertical de una industria emerge como resultado de decisiones optimas de inversión bajo incertidumbre. Las firmas eligen entre integrarse verticalmente, invertir en establecer relaciones con proveedores de insumos especializados o simplemente comprar insumos estandarizados. Dicho marco teórico provee un contexto natural para abordar diversas cuestiones relevantes: Porque las relaciones verticales varían entre industrias así como entre firmas dentro de cada industria? Porque no todas las empresas grandes se integran verticalmente? Cuáles son los efectos de cambios en las propiedades del proceso estocástico que gobierna los shocks a nivel firma sobre la estructura vertical de una industria? Encontramos que mayor incertidumbre está asociada con mayor probabilidad de tercerización de la producción de insumos; firmas verticalmente integradas son más grandes y más eficientes; firmas productoras del bien final que son idénticas en tamaño pueden diferir en su estructura vertical; e idénticas firmas verticalmente integradas pueden evolucionar diferencialmente (desintegrarse o permanecer integradas). También se analizan los efectos que cambios en los costos de integrarse tienen sobre el bienestar, y la producción y productividad agregada.

En el segundo capítulo, **“Dual Employment Protection Legislation and the Size Distribution of Firms”** (*con Andrés Erosa*), se desarrolla un modelo de dinámica de firmas con fricciones de búsqueda y costos de despido asimétricos para los trabajadores temporarios y permanentes (legislación dual de la protección en el mercado laboral). Se caracteriza la composición de equilibrio de firmas con diferente crecimiento de productividad a lo largo de su ciclo de vida, y se estudian los efectos de la legislación dual sobre la distribución de tamaño y productividad de las firmas. Los resultados indican que un mayor costos de despido para los trabajadores permanentes juega un rol similar al de un impuesto sobre las empresas grandes y un subsidio sobre las empresas chicas (similar efecto al de políticas que dependen del tamaño de las firmas, "size-dependent policies") distorsionando la selección de firmas y la asignación de recursos entre firmas. De esta manera cambios en la regulación del mercado de trabajo que inducen un mayor empleo de trabajadores temporales genera una caída en la productividad total de los factores. Consistentemente con la evidencia presentada en el actual trabajo de investigación, a pesar de observar similares niveles de productividad por tamaño de firma, países con legislación dual en el mercado de trabajo que incentivan el uso de contratos temporales presentan una mayor fracción de firmas pequeñas (que concentran a su vez una mayor proporción del empleo total) y menor productividad agregada. En este sentido, el modelo brinda una nueva perspectiva sobre los determinantes de las diferencias en la distribución del tamaño de firmas entre países.

En el tercer capítulo, **"Size-Dependent Policies and Vertical Integration"**, motivado por evidencia empírica reciente que indica que los países en desarrollo tienen menos firmas verticalmente integradas y que dichas firmas son más productivas y grandes, se desarrolla un modelo dinámico con firmas heterogéneas que interactúan como compradoras y vendedoras de insumos, con decisiones endógenas de integración vertical, y fricciones de mercado. En este modelo la estructura vertical de las firmas resulta de decisiones óptimas de inversión bajo incertidumbre. Luego se calibra el modelo a la industria manufacturera de Estados Unidos para cuantificar el impacto de políticas tamaño dependientes ("size-dependent policies") en la asignación de recursos entre firmas determinando diferencias en la productividad agregada. Dichas políticas son modeladas como impuestos sobre la producción y el empleo de mano de obra que las firmas con escala por encima de un cierto nivel deben pagar. Las distorsiones sobre el nivel de producción y empleo generan una reasignación de recursos (trabajadores) desde firmas grandes a firmas pequeñas y actúan como una barrera a la integración vertical. Los resultados indican que un impuesto del 5% a la producción sobre firmas con productividad por encima de la media genera una caída en la fracción de firmas verticalmente integradas desde un 8.7% a un 7%, una caída de la productividad agregada del 2.9%, y un incremento del 11% en el tamaño medio de las firmas. Un impuesto al empleo de mano de obra del 15% genera una caída en la fracción de firmas verticalmente integradas desde 8.6% a 7.1%, una caída de la productividad agregada del 2.4% y una caída del tamaño medio de firma del 1.2%.

Dissertation Abstract

In my thesis, **"Essays on Firm Dynamics"**, I develop different models of firm heterogeneity and firm dynamics to investigate the determinants of vertical integration across industries and across firms within industries, as well as the impact of size-dependent policies and dual employment protection legislation on the size distribution of firms and aggregate productivity

In the first chapter, **"Outsourcing versus Vertical Integration: A Dynamic Model of Industry Equilibrium"**, motivated by the empirical fact that vertically integrated producers are more productive, bigger and are matched to better suppliers (with high productivity and size), I develop a dynamic stochastic model of an industry with heterogeneous firms interacting as buyers and sellers, and market frictions that induce a hold-up problem to the manufacturers to account for these facts. In the model economy, an industrial structure emerges as the result of optimal investment decisions that firms undertake under uncertainty. Firms choose whether to integrate, link to external sellers or buy inputs in the market. This theoretical environment provides a natural framework to answer several questions: Why do supply relations vary across industries and across firms within industries? Why aren't all large firms vertically integrated? How do changes in the properties of uncertainty at firm level determine differences in the vertical structure of an industry? We find that higher uncertainty is associated with higher likelihood of outsourcing; vertically integrated firms are larger and more efficient; otherwise identical downstream firms may differ in their vertical structure, and those that are vertically integrated can end up disintegrated or remain integrated. We also analyze the effects of changes in costs of vertical integration and outsourcing on welfare, aggregate output and productivity.

In the second chapter, **"Dual Employment Protection Legislation and the Size Distribution of Firms"**, *(joint paper with Andrés Erosa)*, we develop a theoretical model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers (dual employment protection legislation). We characterize the equilibrium labor composition that firms with different productivity growth rate choose over their life cycle, and we study the effect of dual employment protection legislation on the distribution of firms' size and productivity. The results indicate that high relative firing costs for permanent workers play similar role as a tax to big firms and a subsidy to small firms (size-dependent-policies) by distorting firm selection as well as the allocation of resources across firms. Therefore, changes in the regulation that induce a higher use of temporary workers generate a decline in the level of TFP. Consistent with the evidence also documented in this paper, in spite of having similar labor productivity by firms' size-classes, countries with dual employment protection legislation that incentives or extend the use of temporary contracts have relatively smaller firms (that concentrate a higher fraction of employment), and lower aggregate productivity. In this sense the model gives new insights into the sources of the considerable differences in the firm-size distributions across countries.

In the third chapter, **"Size-Dependent Policies and Vertical Integration"**, motivated by the fact that developing countries have fewer vertically integrated firms than more developed countries; and vertically integrated firms are more productive and bigger, I develop a dynamic model of an industry with heterogeneous firms interacting as buyers and sellers of inputs, endogenous vertical integration, and market frictions. In the model economy, the vertical industrial structure emerges endogenously as the result of optimal investment decisions that firms undertake. I calibrate the model to the US economy to quantify the impact of size-dependent policies on the reallocation of resources across firms determining differences in total factor productivity. Size-dependent policies are modeled as taxes on output or labor costs that establishments above a certain size have to pay. Distortions on production and employment generate a reallocation of resources (employment) from big firms to small firms and act as barriers to vertical integration. I find that a 15% output tax on firms that are above mean level of productivity generates a decline in the fraction of vertically integrated firms from 8.7% to 7%, a decrease in TFP of 2.9%, and an increase of 11% in the mean size of firms. A 15% tax on employment generates a decline in the fraction of vertically integrated firms from 8.6% to 7.1%, a decrease in TFP of 2.4% and a decline of 1.2% in the mean size of firms.

Contents

Chapter 1

Outsourcing versus Vertical Integration: A Dynamic Model of Industry Equilibrium

1.1 Introduction

The organization of economic activity has been a field of extensive research in economics. This literature, which goes back to the seminal paper by Coase (1937), has focused on the scope of the market versus the firm. Since then, important contributions on transaction cost economics and contract theory have been emphasizing the role of transaction costs, asset specificity, supply uncertainty, incomplete contracting, market power and regulation on vertical integration.[1] These models, however are silent about firm dynamics. This is in contrast with new evidence, by Hortaçsu and Syverson (2009), which shows that there is a close relationship between the vertical structure of firms and key determinants (size and productivity) of the dynamic behavior of producers. In particular, vertically integrated producers are more productive, bigger and are matched to better suppliers (with high productivity and size). Similarly, there is a large empirical and theoretical literature on firm dynamics studying size distribution of firms, turnover, mobility and productivity, among

[1] The literature, at the broadest level, has considered the following perspectives on vertical integration: agency theory articles include Alchian and Demsetz (1972) and Holmstrom (1982); transaction costs theory research includes Williamson (1979); and the references for the property right theory are Grossman and Hart (1986) and Hart and Moore (1990). Gibbons (2005) provides a summary and a comparison of these theories. The most recent surveys include Joskow (2005) and Lafontaine and Slade (2007). Recent theoretical and empirical research on the study of the determinants and effects of vertical integration within and across industries include McLaren (2000), Grossman and Helpman (2002), Antras (2003), Acemoglu et al. (2004) and (2005), Novak and Stern (2007a,b), Ciliberto and Panzar (2009), Legros and Newman (2009) and Gibbons, Holden and Powell (2010).

other issues.[2,3] Given the lack of data, however, this literature has abstracted from the vertical relations firms optimally choose. This is the gap the current model tries to fill.

Introducing endogenous vertical structure decisions (i.e. vertical integration versus outsourcing) into industry equilibrium has implications for key variables of interest, such as size distributions, turnover, etc. For example, vertical integration (we refer to it as VI), in contrast with outsourcing, allows firms to avoid hold-up problems, transactions costs, and cost fluctuations; and insure specialized input procurement, but also increases managerial costs. Thus, differences in costs and benefits in VI across industries may have an impact on firms' profitability and survival, determining differences in size distribution of firms and average productivity of an industry.

This paper builds a long-run dynamic entry and exit equilibrium model of heterogeneous upstream (suppliers) and downstream (manufacturers) firms and market frictions that induce a hold-up problem to the manufacturers. Firms choose whether to integrate, link to external sellers or buy inputs in the market. An industrial structure is the result of optimal investment decisions that firms undertake under uncertainty. In this environment, we seek to understand the determinants of the new stylized facts characterizing the vertical relations of firms. Several questions naturally arise in this environment: Why does the share of vertically integrated firms differ across industries and across firms within industries? How is the vertical structure of firms and industries endogenously determined? What are the implications of firms' vertical structure on the size distribution of firms, the turnover and value of firms? Why aren't all large firms vertically integrated? How do changes in the stochastic process (i.e. persistence) governing the uncertainty at firm level determine differences in the vertical structure of an industry (i.e., the share of vertically integrated firms)?

Our results show that, consistent with the facts presented by Hortacsu and Syverson (2009), vertically integrated firms are larger and more productive. Furthermore, more productive manufacturers tend to integrate with more productive suppliers.

[2] Empirical research documents stylized facts on entry, exit, growth, and the size distribution of firms: Mansfield (1962); Dunne, Roberts and Samuelson (1988) and (1999a,b); Davis and Haltiwanger (1992); Sutton (1997); Caves (1998); Bartelsman, Scarpetta, and Schivardi (2003); Bartelsman, Haltiwanger and Scarpetta (2004); Axtel (2001); Foster Haltiwanger and Kirzan (2001); Cabral and Mata (2003); Cooper and Haltiwanger (2006); Foster, Haltiwanger and Syverson (2008); Bernard, Redding and Scott (2009); and Hsieh and Klenow (2009); among others.

[3] The theoretical work on industry dynamics tries to provide interpretations of the observed heterogeneity across individual producers: Simon and Bonini (1958); Lucas (1978); Jovanovic (1982), Hopenhayn (1992 a,b); Ericson and Pakes (1995); Pakes and Ericson (1998); Cooley and Quadrini (2001); Melitz (2003); Albuquerque and Hopenhayn (2004); Klette and Kortum (2004); Clementi and Hopenhayn (2006); Luttmer (2007); Asplund and Nocke (2007), Rossi-Hansberg and Wright (2007); Hopenhayn and Vereshchagina (2009); and Chatterjee and Rossi-Hansberg (2011); among others.

The productivity process of the manufacturers as well as the cost of vertical relations play a key role in the model. We show that when the productivity shocks for manufacturers are less persistent, i.e. there is more uncertainty, the fraction of vertically integrated manufacturers decline. This is consistent with the evidence provided by Kranton and Meinhart (2000). Hence the observed difference in the level of idiosyncratic risk across industry, as documented by Castro, Clementi and MacDonald (2009), are likely to play an important role in vertical relations within industries.

The current paper is related to two literatures. First, it introduces vertical relations into industry dynamics models (see Hopenhayn 1992 and Hopenhayn and Rogerson 1993). Second, it is related to recent papers that study how different organizational forms might emerge as optimal decisions by the firms. In particular, McLaren (2000) and Grossman and Helpman (2002) propose frameworks of incomplete contracting in which final-goods manufacturers decide whether to outsource production of intermediate goods or produce them in-house. The key factor determining the organizational structure is the externality effect yielding the thickness of the market for inputs: The more other final goods manufacturers choose to outsource productions of intermediate goods, the more attractive it becomes for one manufacturer to do so as well. These papers, however, consider homogenous producers who decide on their vertical relations within a static environment without any shocks.

1.1.1 Facts on Vertical Integration

Hortaçsu and Syverson (2009) show that VI status is related to differences in establishment types for the U.S economy.[4] As Table 1 shows, vertically integrated establishments are larger on average. Between 1977 and 1997 vertically integrated plants constitute relatively small fraction, 8 to 9.5 percent, of all establishments of the economy (row 4). Focusing only on multi-unit establishments, vertically integrated plants account for roughly 35 to 40 percent of these multi-unit businesses (row1/row2). Despite their modest share of the overall number of establishments, vertically integrated businesses account for a much larger employment share, 25-30 percent, and roughly half of multi-unit employment (last row).

[4] In order to state if a firm is VI first they determine the industry affiliation of every establishment in the Economic Census (EC), using the Input-Output Industry Classification System (IOIC) by the Bureau of Economic Analysis (EC contains SIC codes so they reclassify it into IOIC). Second, they identify in which industry firms operate. Third, they verify whether any substantial links are present between pairs of industries based on volume trade flows using 1987 I-O Tables: a substantial link exists between an industry A and any other industry if A buys at least five percent of its intermediate materials, or any other industry to which A sells at least 5% of its output. Finally, they find all establishments that the firm owns on both ends of a substantial vertical link and classify them as being vertically integrated.

Table 1: Aggregate Patterns of Vertical Integration, 1977-1997

Non-farm Private Economy

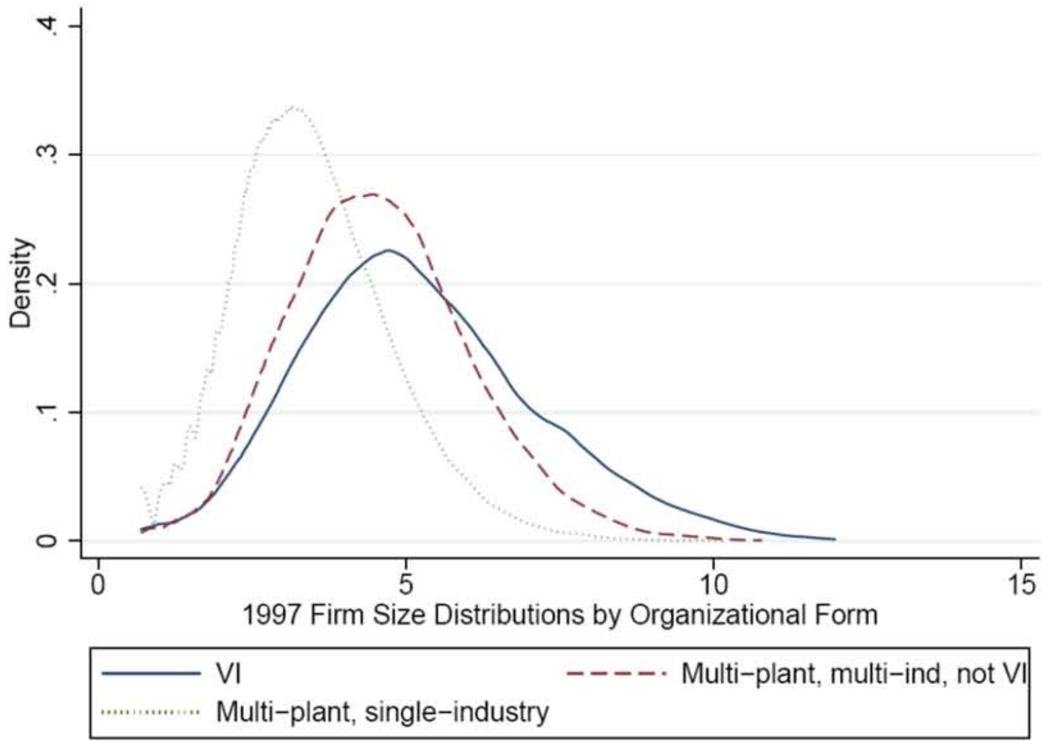
Year	1977	1982	1987	1992	1997
VI Establishments (thousands)	384.3	421.7	546.7	519.8	549.3
Multi-unit establishments (thousands)	1033.7	1167.0	1336.8	1476.6	1605.6
Total establishments (thousands)	4862.2	5049.8	5855.5	6253.2	6831.1
VI establishment share (percent)	7.9	8.4	9.4	8.3	8.0
VI employment (millions)	20.4	21.5	26.9	26.5	28.3
Multi-unit employment (millions)	38.2	42.7	48.3	53.9	60.7
Total employment (millions)	68.1	75.7	87.7	93.6	106.1
VI employment (percent)	29.8	28.4	30.7	28.3	26.7

Source: Taken from Hortaçsu and Syverson (2007).

Therefore, vertically integrated establishments are larger on average than single-unit businesses or non-integrated multi-units. Furthermore, the share of plants that are vertically integrated increases with plants' within-industry size percentiles. While smallest plants in an industry are almost never integrated, 7 percent of the median-sized plant are integrated, and 67 percent of plants in the top percentile of their industry size distribution are integrated.

Figure 1 presents the size densities at firm level. It can be seen that central tendencies are clearly different: vertically integrated firms are the largest on average and their distribution is more skewed. Their size dominates, in first order stochastic dominance (FOSD) sense, to the size of not vertically integrated manufacturers. Notice that there is an overlap among these distributions (firms with the same employment levels have different vertical status).

Figure 1: Firm Size Distributions for Multi-Unit Firms, 1997.



Notes: This figure shows density plots of the firm size distributions (measured by logged total employees) for the three types of multi-establishment firms. See text for details.

Source: Taken from Hortaçsu and Syverson (2009).

Hortaçsu and Syverson (2009) also present a conditional analysis where they regress plant's observables types like size, productivity, and factor intensities (all of them related to plant survival) on an indicator for plants' integration status and a set of control variables (including industry by year fixed effects). The results show that, besides being larger, vertically integrated producers display higher productivity levels (they are on average 40 percent more productive than their unintegrated industry cohorts). Moreover, they investigate why plants have these characteristics and conclude that vertically integrated plants are more productive, larger, and more capital intensive primarily because they were either born into integrated structures that way, or because firms with vertically integrated structures that choose to expand through mergers or acquisitions do so by incorporating existing plants that are also of high-type.

Kranton and Minehart (2000) study the relationship between the vertical structure of firms and idiosyncratic uncertainty in demand, putting special emphasis in a special case of vertical relation, networks (an intermediate level of organization between VI and markets). In the last few decades the importance of input procurement by manufacturer-supplier exchange networks has increased a lot.[5] Therefore, Kranton and Minehart (2000) study the conditions under which industries are likely to be organized as networks.

In their model, manufacturers can decide to build a dedicated asset to produce their own inputs, or they can invest in links to external sellers from which they buy specialized inputs or, alternatively, they get inputs from arm-length markets. The results indicate that there is a connection between industrial structure and uncertainty in demand. Networks appear to be more efficient than vertically integrated structures when uncertainty in demand is substantial: higher dispersion of buyer's idiosyncratic demand shocks should be associated with network-like industrial structures and more connected network structures.

Their result is consistent with several case studies. They cite the case of the US automobile industry in 1920, when there was an increase in uncertainty because of competition from the emerging used-car market and new independent manufacturers. After that, the big automakers Ford and GMC moved away from vertical integration to flexible arrangements with independent suppliers (suggesting that disintegration is a response to underlying environmental uncertainty). The same trend occurred in the film industry in the 1940s, when the volatility in demand for Hollywood movies increased due to the advent of television, and firms moved away from vertical integration to a more flexible system with outsourcing for many aspects of film production.

Summarizing, we want to focus on the following empirical facts documented in Hortaçsu and Syverson (2009) and the main result presented in Kranton and Meinhart (2000):

- *Fact 1)* Vertically integrated plants are larger on average and their size distribution is more skewed.
- *Fact 2)* When vertically integrating, big and efficient downstream firms choose to acquire upstream production units that are also big and efficient.
- *Fact 3)* The fraction of vertically integrated plants increases with the plant's

[5] For example, from 1980 to 1990, the major car manufacturers reduced their number of direct input suppliers by more than 50 percent (Noteboom, 1999). This trend is more prominent in Japanese automobile and electronic manufacturing. The number of direct suppliers to Japanese car manufacturers in 1988 was roughly one half of what it was for American or European manufacturers, for similar volumes of production (Lamming, 1993). For electronics and automobiles, Nishiguchi (1994) presents wide-ranging evidence from Japan on how firms rely more and more on a subset of suppliers with whom they maintain close business ties. In the period from 1980 to 1990, Fuji Electric Tokyo bought an additional 7 percent of its inputs from sub-contractors but it has reduced the number of principal subcontractors by 38 percent. On average, electronic assembly contractors have 3.36 regular costumer each of whom placed orders several times per year.

within-industry size percentiles.

- *Fact 4)* Vertically integrated plants have higher productivity.
- *Fact 5)* When uncertainty in demand is substantial, firms are more likely to invest in links with specific investments (rather than becoming vertically integrated or transact standardized inputs in the market).

1.2 Environment

1.2.1 Key features of the model

We develop a long-run dynamic industry equilibrium model with heterogeneous firms interacting as buyers and sellers of inputs. Final good manufacturers are heterogeneous in their productivity, which is stochastic, denoted by z. They need one unit of input to produce. In order to obtain it they have three options: buy homogeneous inputs from a supplier in the competitive market; build relation with a supplier (i.e., link) to buy a specialized input; or become vertically integrated with a supplier to produce in-house a specialized input. With this unit of input they produce z units of the final good. The final good is homogeneous and is sold in a competitive market.

Suppliers can produce either a standardized (homogeneous) input, or if integrated/linked they can produce a specialized input. When producing a specialized input, suppliers differ in their productivity level, which is denoted by ε. When producing a standardized input, suppliers are homogenous and standardized inputs are sold in a competitive market.

When a manufacturer enters the industry, since it is unattached (it does not have an existing specialized supplier; it is neither VI nor linked), it has to obtain its inputs from the market for standardized inputs. In particular, it pays a price p_s to buy one unit of input. It is assumed that this price is determined by Bertrand competition among unattached suppliers. Once p_s is paid, the manufacturer learns the productivity ε, of the supplier. Given the (z, ε) pair, the manufacturer, if it does not exit the industry, has three options: first, it can simply ignore ε and use the standardized input. In this case the manufacturer simply produces z units of the final good and pays the fixed cost of production (C_f^m). It is assumed that productivity of an unattached supplier is *iid* over time. Next period this manufacturer will start the period in exactly the same situation (as an unattached manufacturer), this is paying C_f^m, buying one unit of input and learning a new z and ε.

Second, given (z, ε), the manufacturer can invest (h) to become linked with the particular supplier (we refer to links as L). In this case the manufacturer produces

z and pays $C_f^m - c(z, \varepsilon)$, where $c(z, \varepsilon)$ represents the cost advantage associated with getting a specialized input from a particular supplier. A manufacturer pays for a specialized input p_s^L, which is determined by Nash Bargaining. As long as the manufacturer and supplier remain linked, ε remains the same. Next period if z remains the same, the pair continues to be linked. If z changes, however, the manufacturer starts next period as an unattached manufacturer (i.e. it has all the same options) with a particular ε at hand (with the same supplier). Finally, the manufacturer can pay $h + P_{VI}$ and become vertically integrated with a particular supplier and produce in-house a specialized input. In this case, it produces z and faces the cost $C_f^m + C_f^{VI} - c(z, \varepsilon)$. Here C_f^{VI} represents the additional cost of being vertically integrated. Once a manufacturer and a supplier become vertically integrated, they continue to do so until z changes upon which manufacturer can reoptimize, although in order to continue vertically integrated the manufacturer does not need to make any investment.

In this framework, once a manufacturer buys form a supplier it cannot switch partner until next period, thus market frictions induce a hold-up problem (as in Grossman and Hart 1986) to linked manufacturers.[6] Moreover, uncertainty plays a key role. Given that under vertical integration manufacturers face a relatively high cost of governance (as in Grossman and Helpman 2002), reflected by a higher fixed cost of production, vertical integration reduces flexibility when facing a negative shock (compared to links and the use of standardized inputs).

Therefore, there is a clear trade-off between links and vertical integration. On the one hand, a linked manufacturer has lower fixed costs but, faces higher endogenous variable costs (determined by the input price negotiation, as it will be explained later on). On the other hand, becoming vertically integrated requires a bigger investment, and imply higher fixed costs, but lower variable costs to manufacturers. From now on, we use the terms manufacturer and downstream firms, as well as suppliers and upstream firms, interchangeably (notice subscripts and superscripts m and s, for manufacturers and suppliers, respectively).

1.2.2 Incumbent firm's problem

We assume that there is no aggregate uncertainty. Thus, by a law of large numbers, all aggregate quantities and prices are deterministic over time, although at the

[6] The hold-up problem is induced by the opportunistic behavior of the supplier. After matching with a given supplier, once the manufacturer has sunk the investment h, there is a bilateral monopoly situation and the supplier seeks to renegotiate the agreement increasing the input price from p_s to p_s^L. This increases the incentives of the manufacturer to buy standardized inputs or become vertically integrated because the manufacturer is not the full residual claimant of the additional returns the investment generates. Anticipating this, the buyer has an incentive to take the supplier into the firm (becoming VI).

firm level, from the point of view of a manufacturer, each firm still faces idiosyncratic uncertainty. We will focus on steady-state stationary equilibrium in which all aggregate variables are constant over time.

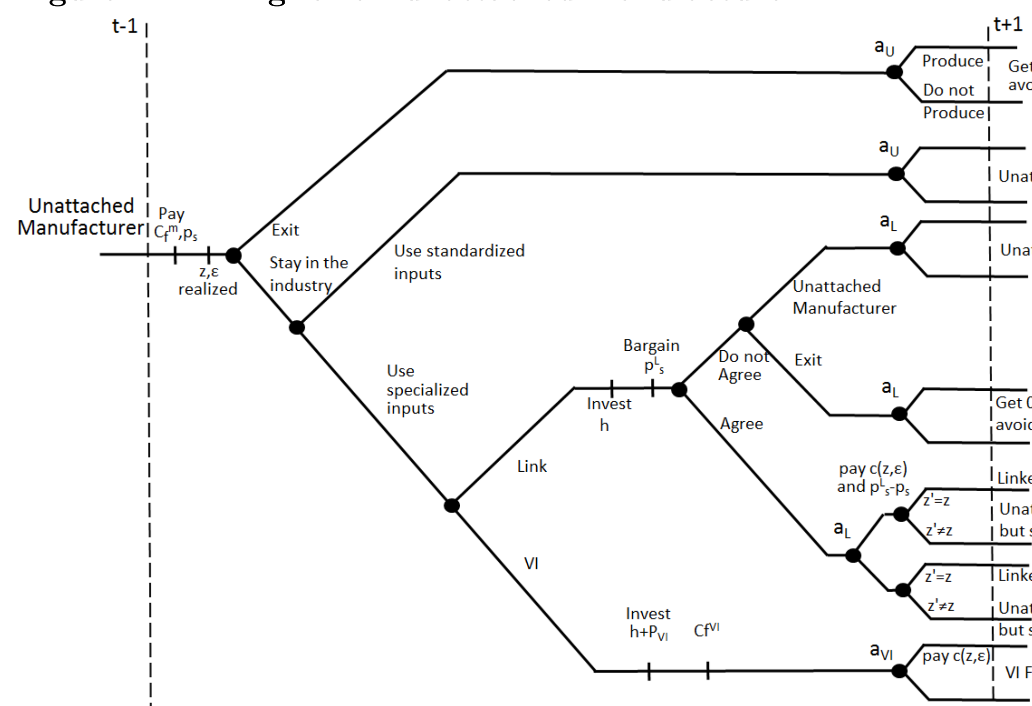
Manufacturers

By using one unit of input, a manufacturer produces a quantity z of homogeneous final goods, where z indicates the manufacturer's managerial ability, and sell the production in a competitive market at a price p. Moreover, we assume that z is independent across firms and follows a Markov process with cdf $F(z'/z)$ and density function $f(z'/z)$. In addition, we assume that F is strictly decreasing in z and $z \in Z$, where $Z = \{z_1, z_2, ..., z_n\}$ and $z_{i+1} > z_i$ for all i. In other words, the higher is the managerial ability of a manufacturer today, the more likely it will be higher tomorrow.[7]

Unattached manufacturer

An unattached manufacturer, at the beginning of every period before the current productivity shock is realized, has to pay a fixed cost of production, C_f^m. In addition, it pays an up-front price, p_s, for the standardized input to a randomly matched supplier. Figure 2 represents the decisions and timing.

Figure 2: Timing for an unattached manufacturer.



[7] As in Hopenhayn (1992a), this assumption implies that expected discounted profits are an increasing function of firm's current productivity shock.

Once C_f^m and p_s are paid, the idiosyncratic productivity shock, z, is realized and the manufacturer learns the quality of the specialized input the supplier can produce. We assume that the supplier's type, ε, has density function $g^s(\varepsilon)$, and $\varepsilon \in E$, where $E = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$ and $\varepsilon_{i+1} > \varepsilon_i$ for all i. As explained before, ε is a match-specific productivity, which can also be interpreted as the managerial ability of the supplier to design and produce a new good and input, and to synchronize production process, together with the matched manufacturer.

Once z and ε are known, the manufacturer decides whether to stay or exit the industry for the next period, and, if it stays in the industry, it must decide whether to use standardized inputs or specialized inputs. In addition, in each situation, it has also to decide whether to produce or not. Thus, if the productivity is very low, in order to avoid paying the fixed costs and the cost of the standardized input, the manufacturers may decide to exit the industry for next period. Therefore, as in standard industry dynamics models, there is endogenous exit, hence, in steady state, there is ongoing entry and exit of manufacturers. If the manufacturer stays in the industry and decides to use standardized inputs it continues as an unattached firm (paying p_s again and learning new values for z and ε).

In order to use specialized inputs the unattached manufacturer has two alternatives, either to become linked with the supplier or become vertically integrated with it (acquire supplier's plant). In both cases, the manufacturer must make specific investments, h (this cost can be thought of as cost in designing a suitable input for the pair z and ε -which is specific to the match- i.e. training costs, costs of providing equipment, know-how, etc.). This investment has two effects, to keep the same supplier's type ε, and to reduce the variable costs to $c(z, \varepsilon)$,. We assume that z and ε are complements. In particular let's assumed that the variable cost function $c(z, \varepsilon)$ satisfies increasing differences.[8,9]

If the manufacturer becomes linked with the supplier, we assume that the reduction in variable costs lasts until z changes, and in that case, in order to take advantage of specialized inputs the manufacturer has to invest again h designing a suitable input for the new pair (z, ε). Moreover, once the specific investment is sunk, the price for the specialized input, $p_s^L(z, \varepsilon)$, is negotiated (determined by Nash Bar-

[8] A manufacturer of type z that is matched with a supplier of type ε has cost advantage $c(z, \varepsilon)$ that satisfies the following property:

$$c(z_i, \varepsilon_j) - c(z_i, \varepsilon_{j-1}) > c(z_{i-1}, \varepsilon_j) - c(z_{i-1}, \varepsilon_{j-1}) \ \forall i, j = 1, .., n$$

[9] The assumptions made on the variable cost function generates a firm behavior which, as it will be shown later on, is in line with new empirical evidence. In particular, Kuglery and Verhoogen (2012), using data from the Colombian manufacturing census, documents that larger plants charge more for their outputs and pay more for their material inputs, and proposes a model of endogenous input and output quality choices by heterogeneous firms to explain the observed patterns.

gaining Solution, NBS). Hence specific investments are subject to hold-up problem which increases the incentives to buy standardized inputs or become vertically integrated, as explained before.[10] Notice that a linked manufacturer firm has the same fixed costs (C_f^m) has lower variable costs ($p_s^L(z,\varepsilon) - c(z,\varepsilon)$) relative to an unattached firm. If the manufacturer decides to become vertically integrated, in addition to the specific investment, h, it has to pay an acquisition price P_{VI} to the supplier (as it will become clear later P_{VI} will correspond to the market value of the supplier). By becoming vertically integrated the manufacturer avoids the hold-up problem.

As in Grossman and Helpman (2002), due to the lack of complete specialization and the extra governance costs associated to managing different plants, we assume that VI increases manufacturer's fixed production costs. This means that a vertically integrated manufacturer has to pay, in addition to the same fixed cost as the standardized manufacturer, C_f^m, and the fixed cost of the acquired supplier, C_f^s, a managerial fixed cost, λ (which is assumed to be positive). Furthermore, notice that uncertainty plays a key role: VI increases firms' fixed costs to $C_f^m + C_f^s + \lambda$ reducing its flexibility when facing a negative shock (when compared to links and market transactions).

When becoming vertically integrated we assume that, in contrast with the link case, the cost advantage $c(z,\varepsilon)$, for different levels of z, is permanent. We assume that, by paying a higher fixed cost of production every period, a vertically integrated firm redesigns the input every time z changes without any additional cost. For the next period, the manufacturer starts as a VI firm.

Therefore, the state variables for an unattached manufacturer are its idiosyncratic productivity, z, and the quality of its supplier, ε. Thus, assuming stationarity (distributions, and thus also prices, do not change over time), the value function for the unattached manufacturer firm is:

$$V^U(z,\varepsilon) = \max_{x'_U \in \{Exit,U,L,VI\}} I_{(x'_U = U, Exit)} V^{UU}(z,\varepsilon) + I_{(x'_U = L)} V^{UL}(z,\varepsilon) + I_{(x'_U = VI)} V^{UVI}(z,\varepsilon), \tag{1.1}$$

where $x'_U : [z,\varepsilon] \rightarrow \{Exit, U, L, VI\}$ denotes the decision rule associated to the vertical relation chosen by the unattached manufacturer, and I is the indicator function given x'_U.

The first term within the max operator in this value function corresponds to the case where the manufacturer remains unattached using standardized inputs in

[10] Since the solution anticipates the hold-up, the specialized input price $p_s^L(z,\varepsilon)$ is the time consistent price for the specialized input (Nash bargaining over present discounted values after h is incurred). In addition, Nash bargaining implies that more productive firms pay more for inputs (consistent with Kugler and Verhoogen (2012)).

current production and decides whether to exit or not for the next period. Formally, the corresponding value function associated to this choice is

$$V^{UU}(z,\varepsilon)=\max_{a_U\in\{0,1\}} a_U pz - p_s - C_f^m + \beta\left\{\underbrace{I_{(x_U'=Exit)}0}_{\text{Exit}},\underbrace{I_{(x_U'=U)}\sum_{z'}\sum_{\varepsilon'}V^U(z',\varepsilon')f(z'|z)g^s(\varepsilon')}_{\text{Unattached (new draw of supplier)}}\right\},$$

where $a_U : [z,\varepsilon] \to \{0,1\}$ is the static production decision rule (the unattached manufacturer decides whether to produce or not in the current period).

The second term within the max operator in Equation (1) corresponds to the situation in which the manufacturer uses specialized inputs by linking with the supplier. The value function for this case is

$$\begin{aligned} V^{UL}(z,\varepsilon) \;=\; & \max_{a_L\in\{0,1\}} a_L[pz-(p_s^L(z,\varepsilon)-p_s)+c(z,\varepsilon)]-p_s-C_f^m-h \\ & +\beta\left[V^L(z,\varepsilon)f(z'=z|z)+\sum_{z'\neq z}V^U(z',\varepsilon)f(z'|z)\right], \end{aligned}$$

thus the manufacturer decides whether to produce or not, $a_L : [z,\varepsilon] \to \{0,1\}$, and negotiates the input price, $p_s^L(z,\varepsilon)$, with the supplier. As long as z remains the same for the next period, the pair continues to be linked (as it can be seen in the first term in the continuation value). If z changes, however, the manufacturer starts next period as an unattached manufacturer with the same previous ε at hand (look at the second term in the continuation value).

The third term within the max operator in Equation (1) represents the value of becoming vertically integrated with the supplier,

$$V^{UVI}(z,\varepsilon)=\max_{a_{VI}\in\{0,1\}} a_{VI}[pz+c(z,\varepsilon)]-p_s-\underbrace{C_f^{VI}}_{C_f^s+\lambda}-C_f^m-(P_{VI}+h)+\beta\sum_{z'}V^{VI}(z',\varepsilon)f(z'|z),$$

where $a_{VI} : [z,\varepsilon] \to \{0,1\}$ is the static production decision rule, and C_f^{VI} is the additional fixed cost of production of a vertically integrated manufacturer, $C_f^{VI} = C_f^s+\lambda$. Thus, the manufacturer decides whether to produce or not and it will start the next period as a vertically integrated firm, that is, with the same cost advantage as a firm that continues linked, but with higher fixed costs of production. By standard dynamic programming arguments (e.g., see Stokey and Lucas (1989)), one can show that there is a unique value function satisfying these Bellman equations. The same applies to the Bellman equations in the next section.

Notice that by becoming a linked firm, the manufacturer faces lower fixed costs (just C_f^m) and higher variable costs ($p_s^L(z,\varepsilon) - c(z,\varepsilon)$) relative to becoming a vertically integrated firm. Besides, by becoming vertically integrated, the manufacturer faces higher fixed costs ($C_f^m + C_f^{VI}$) and lower variable costs (it does not pay p_s and receives the cost advantage $c(z,\varepsilon)$) relative to an unattached manufacturer; and has higher fixed costs ($C_f^m + C_f^{VI}$) and lower variable costs (doesn't pay $p_s^L(z,\varepsilon)$) relative to a linked firm. Thus, there is a clear trade-off of linking versus becoming vertically integrated. We will discuss later on how the properties of the stochastic process (i.e. persistence and variance) governing the uncertainty at firm level also plays a role in these trade-offs, and thus determine differences in the vertical structure of firms across industries.

Linked manufacturer

At the beginning of every period, a manufacturer linked with a supplier of type ε pays a fixed cost of production C_f^m, and productivity z is realized. If the new productivity shock z is equal to the previous shock, then the link continues and firms trade inputs at the same negotiated input price $p_s^L(z,\varepsilon)$ from the previous period and production takes place. Otherwise, if the realization of the new shock z is different from the previous one, the link is broken and the manufacturer has to decide again whether to invest in a link or not. Moreover, if the link is broken, it becomes again an unattached manufacturer, hence it has the same continuation options as an unattached firm (notice that $V^U(z,\varepsilon)$ contains the options of reestablishing the link, becoming VI or using the standardized input), with the only difference that it is matched with the same supplier as in the previous period.

The value function of a linked manufacturer when z has not changed is given by

$$\begin{aligned} V^L(z,\varepsilon) = \quad & pz - p_s^L(z,\varepsilon) + c(z,\epsilon) - C_f^m \\ & +\beta \left\{ V^L(z,\varepsilon) f(z' = z|z) + \sum_{z' \neq z} V^U(z',\varepsilon) f(z'|z) \right\}, \end{aligned} \tag{1.2}$$

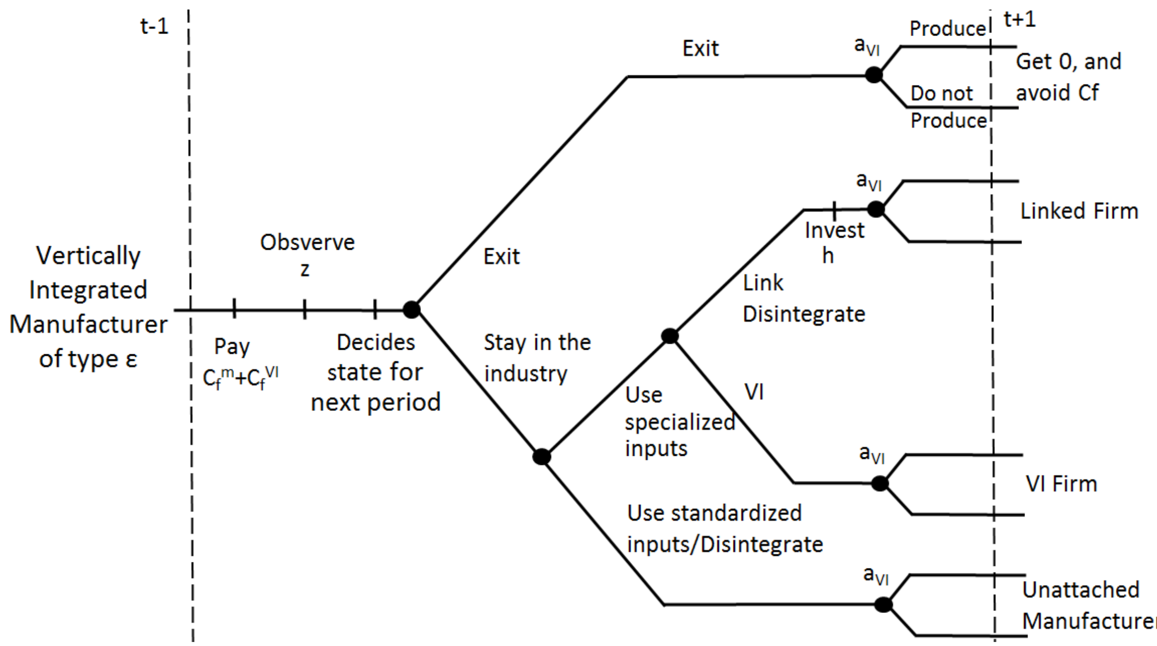
which, after some simple operations, becomes

$$V^L(z,\varepsilon) = \frac{pz - p_s^L(z,\varepsilon) + c(z,\varepsilon) - C_f^m}{1 - \beta f(z'=z|z)} + \frac{\beta(1 - f(z'=z|z))}{1 - \beta f(z'=z|z)} \sum_{z' \neq z} V^U(z',\varepsilon) f(z'|z). \tag{1.3}$$

Vertically integrated manufacturer

A manufacturer that is vertically integrated with a supplier of type ε pays fixed costs of production C_f^m and C_f^{VI}, productivity z is realized (while ε remains the same). Therefore, it decides current production, $a_{VI} \in \{0, 1\}$, and the state for the next period (Figure 3). It has the same continuation options as for the unattached firm (invest in L, get a new supplier, exit the industry), but in order to continue vertically integrated with same supplier it has to make no additional investment. In the case of investing in a link (disintegrate but remain matched with the same supplier) the manufacturer produces today as a vertically integrated firm and, since next period on, it has to pay a negotiated input price $p_s^L(z, \varepsilon)$.

Figure 3: Timing for a vertically integrated manufacturer.



According to the previous timing, the value function for a vertically integrated manufacturer looks like in Equation (4). A manufacturer with productivity z that enters the current period being vertically integrated with a supplier of type ε , after paying the fixed costs, has to decide whether to produce or not so as to maximize the per period profit. Next, it has to decide in which state it enters the next period, this is either continue vertically integrated (with the fourth continuation value), without making any additional investment, or disintegrate. In case it decides to disintegrate, it still has the option to continue producing with the same supplier by becoming linked with it after investing h (with the third continuation value). Finally, it can also become an unattached manufacturer starting the next period with a new ε, or exit the industry.

$$
\begin{aligned}
V^{VI}(z,\varepsilon) \quad = \quad & \max_{a_{VI}\in\{0,1\},x'_{VI}\in\{Exit,U,L,VI\}} a_{VI}[pz+c(z,\varepsilon)] - C_f^{VI} - C_f^m - hI_{(x'_{VI}=L)} && (1.4)\\
& +\beta \left\{ \underbrace{I_{(x'_{VI}=Exit)}0}_{\text{Exit}} + \underbrace{I_{(x'_{VI}=U)} \sum_{z'} \sum_{\varepsilon'} V^U(z',\varepsilon') f(z'|z) g^s(\varepsilon')}_{\text{Unattached}} \right. \\
& \underbrace{+I_{(x'_{VI}=L)} \left[V^L(z,\varepsilon) f(z'=z|z) + \sum_{z'\neq z} V^U(z',\varepsilon) f(z'|z) \right]}_{\text{Link}} \\
& \left. \underbrace{+I_{(x'_{VI}=VI)} \sum_{z'} V^{VI}(z',\varepsilon) f(z'|z)}_{\text{Vertical Integration}} \right\}, && (1.5)
\end{aligned}
$$

where $x'_{VI} : [z,\varepsilon] \to \{Exit, U, L, VI\}$ is the decision rule, that is the state chosen for the next period, and I is the indicator function given x'_{VI}.

Suppliers

Unattached supplier

Unattached suppliers produce one unit of an homogeneous input and compete in prices. They have zero marginal cost and pay a fixed cost, C_f^s, every period. Once they match with a manufacturer, the quality ε of the specialized input they are able to produce is realized. In case they remain as unattached input supplier the quality of the match, ε, is *iid* over time and across suppliers. The value function of an unattached supplier is

$$
\begin{aligned}
W^U(z,\varepsilon) \quad = \quad & \underbrace{I_{(x_U=U)} \left[p_s - C_f^s + \beta \sum_{\varepsilon'} \sum_{z'} W^S(z',\varepsilon') J^m(z') g^s(\varepsilon') \right]}_{\text{produce standardized inputs}} && (1.6)\\
& \underbrace{+I_{(x_U=L)} \left[p_s^L(z,\varepsilon) - C_f^s + \beta W^L(\cdot) f(z'=z|z) + \beta \sum_{z'\neq z} W^U(\cdot) f(z'|z) \right]}_{\text{linked}} \\
& \underbrace{+I_{(x_U=VI)} P_{VI}(z,\varepsilon)}_{\text{VI}} && (1.7)
\end{aligned}
$$

where $x_U : [z, \varepsilon] \to \{Exit, U, L, VI\}$ is the current period decision rule of the unattached manufacturer that is matched with this supplier, and I is the corresponding indicator function given x_U. The function $J^m(z')$ is an equilibrium object that represents the density of manufacturers, for each particular productivity level z, that will be looking for a standardized supplier in the next period. For each value of z the density $J^m(z')$ is determined by the process of entry, exit, investment in links and vertical integration.

Specialized supplier

A specialized (linked) supplier produces one unit of the input using the same technology as an unattached supplier. It offers an input of heterogeneous quality which is permanent over time (as explained before, conditional on producing with the same manufacturer every period). In addition it negotiates the input price in a bilateral monopoly situation with the manufacturer, due to the market frictions (once the manufacturer is matched with a supplier it cannot switch partner until next period).

The value function of a linked supplier is

$$W^L(z, \varepsilon) = p_s^L(z, \varepsilon) - C_f^s + \beta \left\{ W^L(z, \varepsilon) f(z' = z|z) + \sum_{z' \neq z} W^U(z', \varepsilon) f(z'|z) \right\}, \quad (1.8)$$

which, after some simple operations, becomes

$$W^L(z, \varepsilon) = \frac{p_s^L(z, \varepsilon) - C_f^s}{1 - \beta f(z' = z|z)} + \frac{\beta(1 - f(z' = z|z))}{1 - \beta f(z' = z|z)} \sum_{z' \neq z} W^U(z', \varepsilon) f(z'|z). \quad (1.9)$$

We assume that, if a linked manufacturer breaks the link with a supplier, then the supplier returns to the standardized inputs market, gets matched with another unattached manufacturer, and gets a new draw of ε from $g^s(\varepsilon)$. In addition, if a supplier becomes vertically integrated it gets P_{VI} and disappears. Furthermore, if the manufacturer disintegrates, then the supplier appears again as an unattached supplier.

Equilibrium prices for the specialized input and supplier's acquisition price

Given all the previous value functions, we can now define the prices for the specialized inputs and the acquisition price for a supplier firm that a manufacturer pays when vertically integrating. The first one is defined, according to Nash Bargaining, as follows:

$$p_s^L(z,\varepsilon) = \arg\max_{p_s^L} \left[V^L(z,\varepsilon) - \left(\underbrace{V^{UU}(z,\varepsilon)}_{\text{Manufacturer's outside option}}\right)\right]^{\theta} \tag{1.10}$$

$$\left[W^L(z,\varepsilon) - \left(\underbrace{W^{U}(z,\varepsilon)}_{\text{Supplier's outside option}}\right)\right]^{1-\theta}, \tag{1.11}$$

where θ is the bargaining power of the manufacturer. Thus solving for the bargained specialized input price and using the previsously defined value functions we get:

$$\begin{aligned} p_s^L(z,\varepsilon) = \; & (1-\beta f(z'=z|z))(1-\theta)\left[\tfrac{pz+c(\cdot)-C_f^m}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)}\sum_{z'\neq z} V^U(\cdot)f(z'|z)\right. \\ & \left. - \left(pz - p_s - C_f^m + \beta \max\left\{0, \sum_{z'}\sum_{\varepsilon'} V^U(z',\varepsilon')f(z'|z)g^s(\varepsilon')\right\}\right)\right] \\ & -\theta\left[\tfrac{-C_f^s}{1-\beta f(z'=z|z)} + \tfrac{\beta(1-f(z'=z|z))}{1-\beta f(z'=z|z)}\sum_{z'\neq z} W^U(z',\varepsilon)f(z'|z)\right. \\ & \left. - \left(p_s - C_f^s + \beta\sum_{\varepsilon'}\sum_{z'} W^U(z',\varepsilon')J^m(z')g^s(\varepsilon')\right)\right]. \end{aligned} \tag{1.12}$$

Thus, the specialized input price depends only on the value functions of the unattached manufacturer and supplier. Moreover, I assume that an unattached manufacturer which optimally chooses to become vertically integrated makes a take-it-of-leave-it offer to the supplier and pays to him a price P_{VI} that is the present discounted value of being an unattached supplier. This is, we assume that the market value of the supplier is $P_{VI} = \beta E_{z',\varepsilon'} W^U(z',\varepsilon')$.

1.2.3 Free Entry Condition

There is free entry of manufacturers who are ex-ante identical. We assume that manufacturer firms that enter the industry make no specific investment. This means that entrants cannot enter the industry being vertically integrated or linked firms, they just enter as unattached manufacturers.

They must pay a sunk downstream entry cost, $C_e^m \geq 0$, the fixed cost of production, $C_f^m \geq 0$ and they buy one unit of the standardized input paying p_s After that, they draw z from $g^m(z)$ and then match randomly with a supplier according to $g^s(\varepsilon)$. Thus, the value of the expected future discounted profits of a new downstream firm is

$$V_e^m(p, p_s) = \sum_{\varepsilon} \sum_{z} V^U(z, \varepsilon) g^m(z) g^s(\varepsilon). \tag{1.13}$$

In the input industry there is also free entry. Entrants are ex-ante homogeneous producers and enter the input industry as unattached suppliers. They first have to pay a sunk upstream entry cost, $C_e^s \geq 0$, and fixed cost, $C_f^s \geq 0$. After doing so, they earn p_s and match randomly with a manufacturer, according to $J^m(z)$, and their type ε is revealed according to $g^s(\varepsilon)$. Thus, the value at entry for an upstream firm is

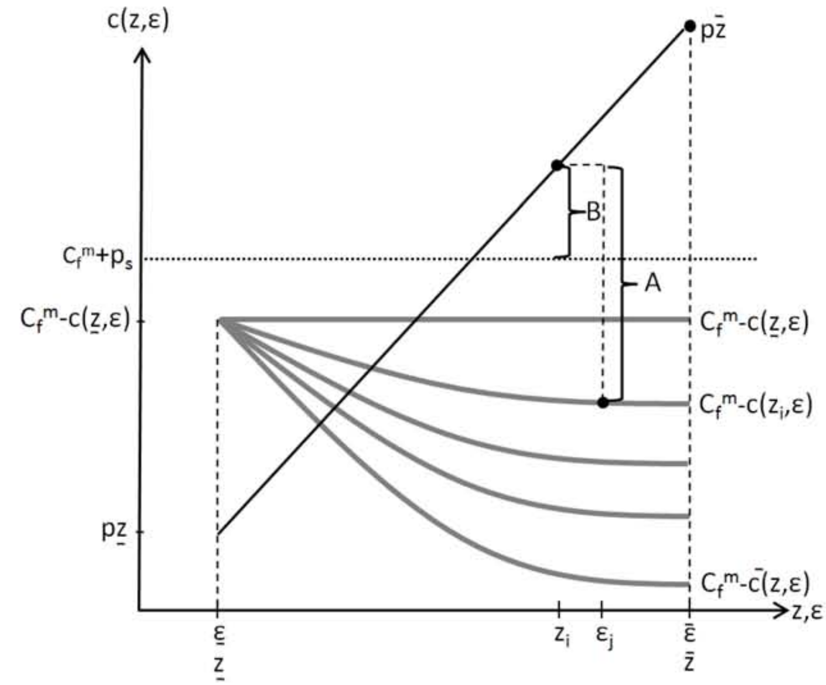
$$W_e^s(p_s, p) = \sum_{\varepsilon} \sum_{z} W^U(z, \varepsilon) J^m(z) g^s(\varepsilon). \tag{1.14}$$

1.2.4 Characterization of Equilibrium

Before defining the stationary equilibrium in this model let's first analyze further the profit function of an unattached manufacturer that reflects some properties of the value function. As mentioned before, we assumed complementarity between manufacturer and supplier's types, in particular we assumed that the variable cost function $c(z, \varepsilon)$ satisfies increasing differences. This means that manufacturers of different types can produce more efficiently with a supplier of high ε-type, but the cost advantage is greater for producers of high z-types. Therefore, assuming a given functional form for $c(z, \varepsilon)$ we can plot $c(z, \varepsilon) + C_f^m$ (solid grey curves in Figure 4) which is weakly decreasing in ε, together with the revenue function, pz, (solid black curve in Figure 4) for an unattached manufacturer. The distance between the latter and $p_s + C_f^m$ is the per period profit of an unattached manufacturer.

The upper curve $c(z, \varepsilon) + C_f^m$ (the straight line) in Figure 4 represents the case of the least efficient manufacturer (denoted by $\underline{z}$). We can see that when it is matched with the most efficient supplier (denoted by $\overline{\varepsilon}$) it does not improve in costs. In contrast, when the most efficient manufacturer (denoted by $\overline{z}$) is matched with the most efficient supplier (denoted by $\overline{\varepsilon}$) there is a big decline in total costs (lower curve).

Figure 4: Costs, revenues and profits of an unattached manufacturer.



Furthermore, for an unattached manufacturer of type z_i matched with a supplier of type ε_j, the static gain from using specialized inputs (net of costs corresponding to the cases of VI or link) is the difference between the distances A and B. Clearly, as it can be seen in the picture, the static gain from using specialized inputs is increasing in z and ε. We will use this property of the profit functions, together with the characteristics assumed on F, to state that the value of investing in the use of specialized inputs is increasing in z and ε.

To gain more intuition about how the model works let's show which vertical structure a manufacturer chooses for next period given the current productivity. In the following proposition we focus on an unattached manufacturer firm, but the same reasoning should be followed for the case of a vertically integrated firm and a linked one:

Proposition 1 There exist a number $z^* > 0$ and a threshold function $\widehat{\varepsilon}(z) : [z_1, z_n] \to [\varepsilon_1, \infty)$ for an unattached manufacturer firm such that:

$$x_U(z,\varepsilon) \in \begin{cases} \{L, VI\} & \text{for all } \{(z,\varepsilon) \in Z \times E : \varepsilon \geq \widehat{\varepsilon}(z)\}, \\ \{Exit\} & \text{for all } \{(z,\varepsilon) \in Z \times E : z < z^* \text{ and } \varepsilon < \widehat{\varepsilon}(z)\}, \\ \{U\} & \text{for all } \{(z,\varepsilon) \in Z \times E : z \geq z^* \text{ and } \varepsilon < \widehat{\varepsilon}(z)\}. \end{cases}$$

Proof Let's first define z^* as the minimum productivity level at which an unattached manufacturer, before observing the current supplier type ε it is matched with, decides to stay in the industry and get a new draw of supplier for next period. Let's compare the two continuations values in $V^{UU}(z,\varepsilon)$, from Equation (1). Given that F is decreasing in z and $c(z,\varepsilon)$ is increasing in z, the continuation value of getting a new draw of ε, $E_{z',\varepsilon'}\left[V^U(z',\varepsilon')/z\right]$, is monotone increasing in z. Therefore, as $V^U(z,\varepsilon)$ is continuous in z, by the intermediate value theorem there exists a thresholds z^* and it is singled valued, and defined as in Hopenhayn (1992):

$$z^* = \inf\left\{z \in Z : \sum_{z'}\sum_{\varepsilon'} V^U(z',\varepsilon')f(z'|z)g^s(\varepsilon') \geq 0\right\}.$$

Now let's look for the set of minimum productivity levels for z and ε at which the value of using specialized goods (becoming vertically integrated or linked) $V^{UL}(z,\varepsilon)$ or $V^{UVI}(z,\varepsilon)$ is greater than or equal to being an unattached manufacturer. We know that for pairs of (z,ε) formed by low values of z and ε, given the assumptions on costs and sunk specific investment, the firm does not decide to become vertically integrated or set up links. Furthermore, in order to have available the continuation values corresponding to VI or L the firm has to invest $h + P_{VI}$ or h, respectively. This means that the corresponding expected future discounted profits plus present revenues must be high enough to recover the costs $h + P_{VI}$ or h. But, given that the continuation values of becoming vertically integrated or linked are monotone increasing in z and ε, and as $V^{UVI}(z,\varepsilon)$ and $V^{UL}(z,\varepsilon)$ are continuous in z and ε, for each value of z, by the intermediate value theorem, there exists ε (a threshold), which is singled valued, at which the unattached manufacturer decides to become vertically integrated or linked. Then let's define a correspondence $\widetilde{\varepsilon}(z)$ that maps values of z into values for ε, $\widetilde{\varepsilon}(z) : Z \to [\varepsilon_1, \infty)$. Thus $\widetilde{\varepsilon}(z)$ is formally defined as

$$\widetilde{\varepsilon}(z) \equiv \left\{\varepsilon \in [\varepsilon_1,\infty) : \text{given } z \in Z,\ \max\left\{V^{UL}(z,\varepsilon), V^{UVI}(z,\varepsilon)\right\} \geq V^{UU}(z,\varepsilon)\right\}.$$

Then, let's define a function $\widetilde{\widetilde{\varepsilon}}(z) : [z_1, z_n] \to [\varepsilon_1, \infty)$ as

$$\widetilde{\widetilde{\varepsilon}}(z) \equiv \left\{\vartheta \in [0,\infty) :\ \vartheta \equiv \inf\left(\widetilde{\varepsilon}(z)\right)\right\}.$$

Then as the function $\widetilde{\widetilde{\varepsilon}}(z)$ is continuous and monotone decreasing in z there exists a threshold z^{**} such that $\widetilde{\widetilde{\varepsilon}}(z^{**}) = \varepsilon_n$, where ε_n is the minimum $\varepsilon \in E$. Then we can define a function $\widehat{\varepsilon}(z) : Z \rightarrow [\varepsilon_1, \infty)$ as

$$\widehat{\varepsilon}(z) \equiv \left\{ \begin{array}{ll} \widetilde{\widetilde{\varepsilon}}(z) & \text{for } z \geq z^{**} \\ \infty & \text{for } z < z^{**} \end{array} \right\}.$$

Therefore, for all $(z, \widehat{\varepsilon}(z)) \in Z \times E$ a downstream unattached manufacturer firm decides to become vertically integrated or have links with the supplier.■

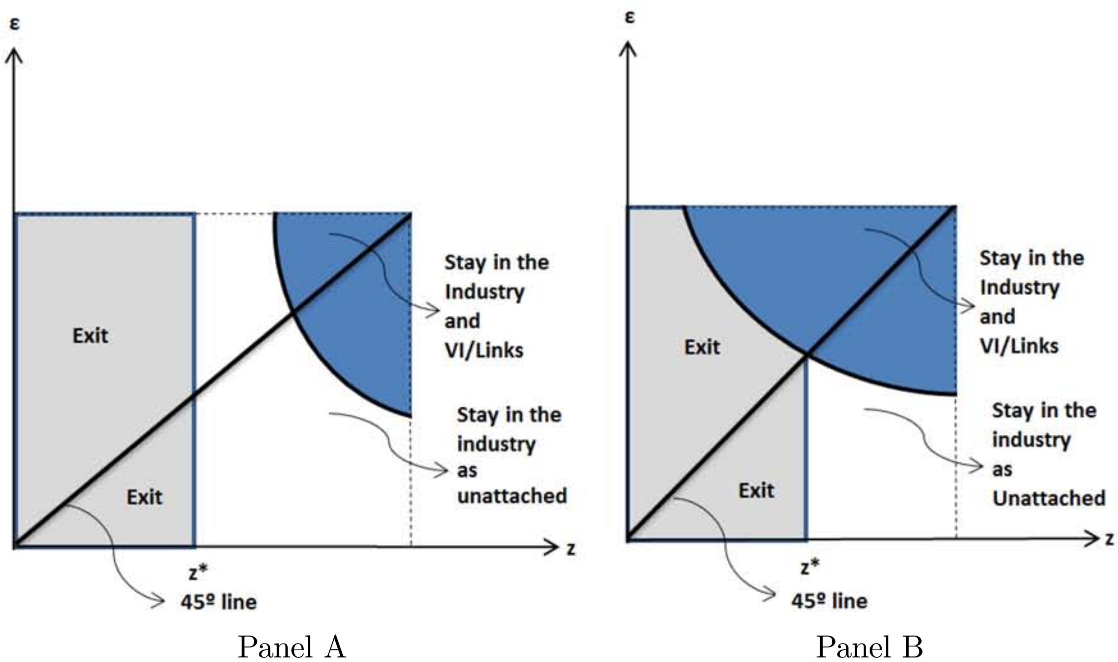
Basically the next proposition states that if an unattached manufacturer firm with a given productivity pair (z, ε) decides to become vertically integrated or linked, then any firm with higher efficiency levels (z, ε) will also become vertically integrated or linked.

Intuitively the previous proposition is a characterization of the decision rule for an unattached manufacturer. It states that, under the assumptions made on costs, this decision rule look like presented in Figure 5. In the horizontal axis we have the productivity of the manufacturer and in the vertical axis the productivity of the supplier. The figure shows the regions of (z, ϵ) under which an unattached manufacturer decides to exit the industry, to become vertically integrated, to set up link or to continue unattached for next period.

In panel A we have the case in which $z^* \cap (z, \widetilde{\varepsilon}(z)) = \varnothing$ for all $(z, \widetilde{\varepsilon}(z)) \in Z \times E$, thus there is only one relevant threshold (z^*) that manufacturers consider to exit the industry. This is, a manufacturer with a productivity shock bellow z^* decides to exit the industry independently to which supplier's type it is matched with. If its productivity level z is above that threshold, the firm decides to remain active in the industry, and if it is matched with an efficient supplier it decides to become vertically integrated or linked.

In panel B we have the case in which $z^* \cap (z, \widetilde{\varepsilon}(z)) \neq \varnothing$ for all $(z, \widetilde{\varepsilon}(z)) \in Z \times E$, thus there is a set of relevant thresholds that manufacturers consider to exit the industry. Furthermore, in contrast with Panel A, a manufacturer with a productivity shock bellow z^* can survive if it is matched with an efficient supplier. The equilibrium shape of the set of relevant thresholds will depend on the parametrization of the model. We will focus on that in the calibration section.

Figure 5: Decision rule for an (z, ε)-unattached manufacturer for next period.



1.2.5 Stationary Equilibrium

The stationary equilibrium definition is standard (for a detailed formal definition see the appendix): A stationary equilibrium in this model is a list of value functions and policy functions for manufacturers and suppliers, prices, invariant measures of firms, and a mass of entrants such that given the prices the policy functions solve the firms' problem, the free entry conditions and the market clearing conditions are satisfied, and the stationary measures of firms are fixed points. In the appendix, it is also explained the algorithm used to compute the equilibrium.

1.3 Quantitative Analysis

1.3.1 Calibration-Preliminary Results

To solve the model numerically, we need to specify functional forms for the demand and firms technology and assign parameter values. Basically, we calibrate our model so that the industry stationary equilibrium matches selected characteristics of the U.S. manufacturing sector taken from the U.S. Census Bureau and from Hortaçsu and Syverson (2007 and 2009). Table 2 summarizes the values for the parameters set a priori.

Table 2: Parameters set a priori

Parameters	Definition	Value	
θ	Bargaining power of the buyer	0.5	assumed
β	Discount factor	0.96	assumed
η	Inverse of demand elasticity	1.164	Nicholson (1989)
ρ	Autoregressive parameter	0.93	Hopenhayn and Rogerson (1993)

Manufacturers and suppliers are assumed to have the same bargaining power, $\theta = 1/2$. In addition, we set a discount factor value $\beta = 0.96$ consistent with a 4% interest rate. We assume a constant elasticity of demand, $p = Q^{-\eta}$, where Q is the aggregate production and η is the inverse demand elasticity which we take equal to 1.164.[11]

We assume that shocks z has lognormal distribution and follows an $AR(1)$ process,

$$ln\ z_t = \delta + \rho \ln\ z_{t-1} + \xi_t, \quad \text{with } \xi_t \sim N(0, \sigma_\xi^2),$$

where ξ_t is the *iid* shock, and the parameter ρ is a measure of persistence of the idiosyncratic productivity process. Changes in the persistence of the shocks will have an impact on how a firm decides its vertical structure given the properties of the costs. Therefore, if persistence is very high, then, loosely speaking, an efficient firm expects that high shocks today will be around for a long time. Conversely, if shocks are not very persistent, then the manufacturer will take into account the possibility of incurring high losses (due to high fixed costs) or not recovering the irreversible investment $(h + P_{VI})$, because there is a strong possibility that they will be incurred relatively soon.

[11] We take the average of the elasticity values published in Nicholson (1989): Food 0.21, Medical Services 0.20, Automobiles 1.20, Housing (Rental) 0.18, Housing (Owner-Occupied) 1.2, Gasoline 0.54, Electricity 1.14, Giving to Charity 1.29, Beer 1.13, Marijuana 1.5.

A 25-points grid was assumed for both discretized shocks z and ε, where we assume $Z = E$ to simplify.[12] The transition matrix for z was obtained by Tauchen's method which approximates the previous $AR(1)$ process for the idiosyncratic shocks. The estimation of its persistence parameter ρ was taken from Hopenhayn and Rogerson (1993), assuming that firms in both models are hit by the same stochastic idiosyncratic productivity process[13]. We took the invariant distribution of the Markov chain matrix for z as the initial distribution $g^m(z)$ and as $g^s(\varepsilon)$.

With respect to the function $c(z, \varepsilon)$ we assume a function as follows

$$c(z_i, \varepsilon_j) = T_1 \left(\frac{z_i - z_1}{z_n - z_1} \right)^{\alpha} \left(\frac{\varepsilon_j - \varepsilon_1}{\varepsilon_n - \varepsilon_1} \right)^{1-\alpha} + T_2,$$

which is increasing in z_i and ε_j, with $\alpha \in [0, 1]$. The parameter T_1 is the maximum gain from searching a supplier, for the most efficient manufacturer (being z_n and matched with an ε_n supplier reduces the nonsunk cost T_1); and T_2 is the gain from investment (by investing $h + P_{VI}$ in becoming vertically integrated, or h in becoming linked, the manufacturer reduce the nonsunk cost in this amount T_2, independently on the type of the supplier it is matched with). The parameter α indicates how important is the manufacturer's type in the decline of variable costs when investment in links and VI take place. Notice that $c(z, \varepsilon)$ is flexible, in the sense that it allows for the absence of increasing differences.

Table 3 presents the value for the calibrated parameters with the corresponding moments the model tries to match. Figure 6 shows the shape of the function $c(z_i, \epsilon_j)$ for the parameter values presented above.

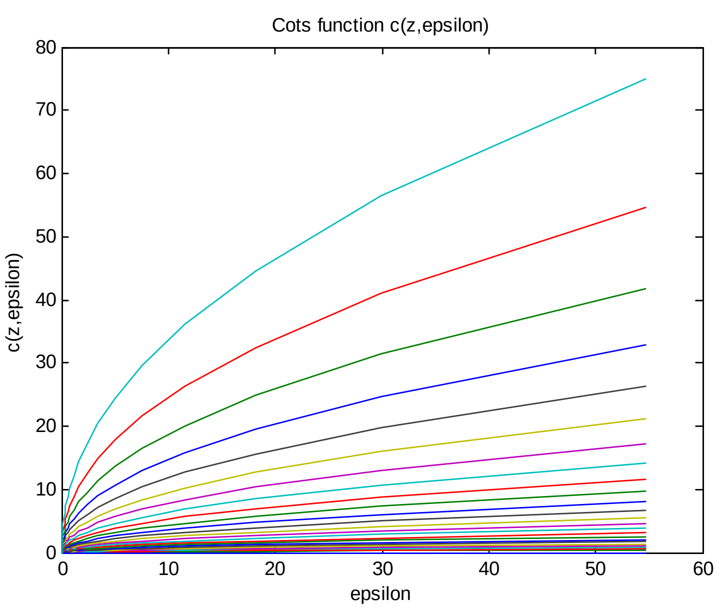
[12] The number of grid points was selected so as to have a smooth enough behavior of firms' decisions.

[13] One could also assume that, under a Leontieff production function, employment follows the same stochastic process as revenues.

Table 3: Calibrated Parameters and moments to fit.

	Definition		Target	Definition
δ	Autoregressive intercept	0	revenue distrib. of firms	
σ_ξ^2	Standard deviation of μ.	0.15		
T_1	Gain from searching for high ε	75		
T_2	Cost reduction	0		
C_f^m	Fixed cost	0.40	10%	Annual exit rate (Bartelsman, Haltiwanger and Scarpetta 2000)
λ	Extra managerial fixed cost of a vertically integrated firm	3.15	$8\% - 9\%$	%VI firms (Hortaçsu and Syverson 2009)
h	Investment cost of L	1.3	25%	% L firms (Uzzi 1996)
α	Relative weight of z in cost reduction	0.47	7%	times the median-sized manufacturing plant is VI (Hortaçsu and Syverson, 2009)
C_e^m	Sunk cost of entry	3.01	$V_e^m(1)$	Entry value at $p = 1$

Figure 6: Cost function $c(z, \epsilon)$



The value of the intercept, δ, and the variance of the error term, σ_ξ^2, of the AR(1) stochastic process for z, as well as T_1 and T_2 are chosen so as to fit the size (revenue) distribution of firms of the US manufacturing sector. Revenue values in the model are expressed in millions of dollars. In particular, we use the U.S. Census Bureau tabulated data prepared by the Small Business Administration (SBA) for year 2002.

Table 4 indicates a mean revenues for all firms of 11,434 millions of dollars. In addition, the share of firms in the first interval of revenues (0-0.99) of 51.45%, and the shares of firms with revenues between (1-4.99), (5-9.99) and (10-49.99) are 22.7%, 5.7% and 7.5%, respectively. Finally, the share of the biggest firms that have revenues above 50 millions is 12.6%. Hence we choose δ, σ_ξ^2, T_2 and T_2 in order to minimize the Euclidean distance between the data and model densities of firms in each scale interval so as to generate a revenue distribution that is in line with Table 4.

Table 4: Size (revenue) distribution of firms

	Receipt Size of Manufacturing Establishments (in millions of dollars)					
	Total	0-0.99	1-4.99	5-9.99	10-49.99	50+
Establishments	344,341	177,099	78,026	19,774	25,893	43,549
		51.4%	22.7%	5.7%	7.5%	12.6%
Receipts ($000)	3,937,164,576	56,607,235	173,543,614	122,826,132	361,399,818	3,222,847,777
		1.4%	4.4%	3.1%	9.2%	81.9%
Mean	11,434					

Source : Based on Census Bureau 2002 tabulated data prepared by the SBA.

The fixed cost C_f^m is selected to fit an exit rate of 10% (taken from Bartelsman, Scarpetta and Shivardi, 2003). Given a normalized final good price $p = 1$, given the value function $V^U(z, \varepsilon)$, the level for the sunk entry cost C_e^m was selected such that $C_e^m = \sum_\varepsilon \sum_z V^U(z, \varepsilon) g^m(z) g^s(\varepsilon)$. In addition, the value for fixed cost, C_f^s, as well as the entry cost, C_e^s, of suppliers were assumed to be equal to the fixed cost and entry cost of manufacturers, respectively.

The extra managerial cost for a vertically integrated manufacturer, λ, and the investment cost, h, were chosen to match a share of 8 to 9 % of vertically integrated firms and a share of linked firms 25%, respectively.[14] When the relative weight of z

[14] Uzzi (1996) studies the Women's Dress industry where manufacturers and contractors are linked by long-term ongoing relationships. He finds that about 25 percent of the manufacturers have networks composed of 5 or fewer exchange partners; 30 percent have exchanges with 5 to 12 partners, while about 40 percent maintain business ties with more than 20 contractors. We take a value of 25% for our calibration given that in our model each manufacturer is supplied with just one supplier. Notice that the exercise

in the variable cost advantage α increases then the chance to become VI or linked for firms with a low z productivity increases. In other words, for firms with a low z and a high ε, the variable cost advantage $c(z, \varepsilon)$ increases with α. Therefore the share small firms that become VI increases with α. Thus, the value for α was chosen so as to fit the percentage of median sized manufacturing plants that are vertically integrated.[15] Table 5 shows the calibration results. It can be seen that the annual exit rate, the share of vertically integrated and linked firms, and the percentage of vertically integrated plants in the median-sized plants are well fitted, while the fit of the size distribution of firms can be improved (Figure 7).

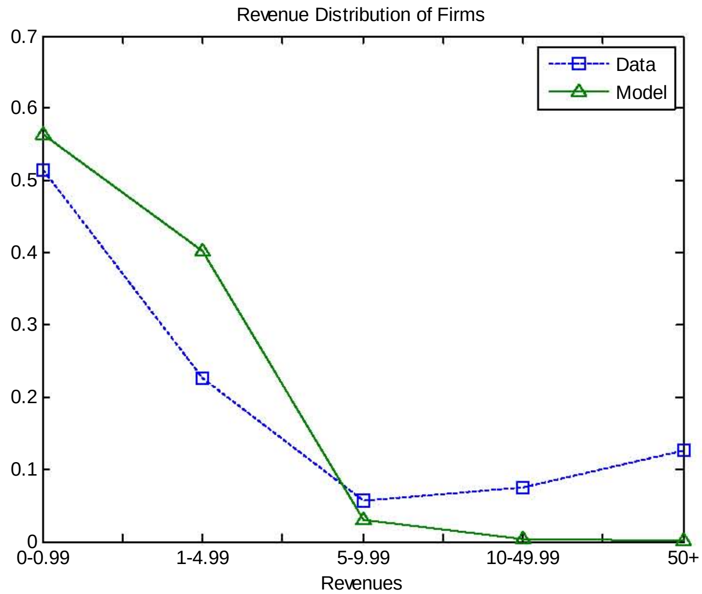
Table 5: Data moments and model moments.

	Model	Data
Share of firms by size		
(revenues in millions of U.S. dollars)		
0-0.99	56.4%	51.4%
1-4.99	40.2%	22.7%
5-9.99	3.1%	5.7%
10-49.99	0.4%	7.5%
50+	0.0%	12.6%
Annual exit rate	8.6%	10%
Share of Linked firms	25.7%	25%
Share of vertically integrated firms	8.4%	8%-9%
Share of vertically integrated median-sized firms	5.2%	7%

we will perform in the following section is to decrease the persistence of the z shocks and look at what happen with the number and share of VI and L firms. And the value of the H's parameters determines the sensitivity of the decision rules to the persistence of z.

[15] The share of VI plants, as well as the percent of the median-sized plants that are integrated, were taken from Hortaçsu and Syverson (2009), as exposed in the introduction.

Figure 7. Size distribution of firms.

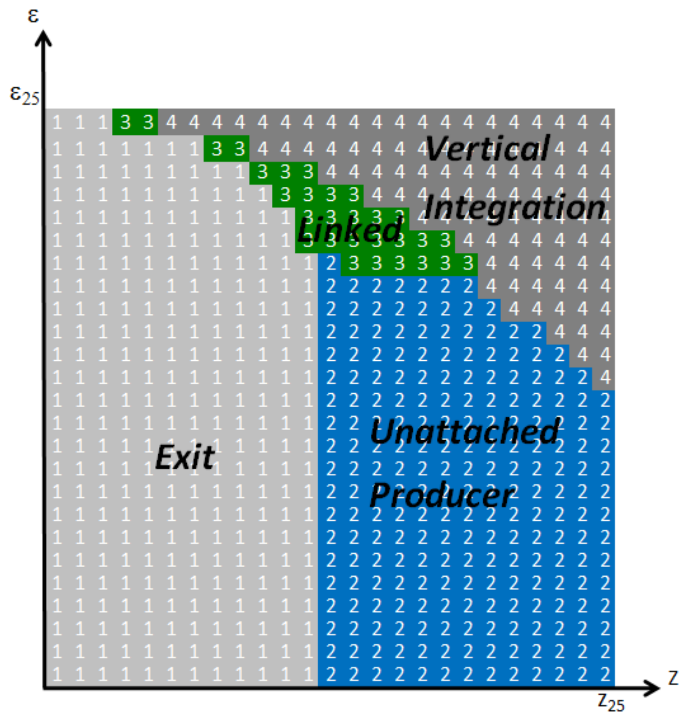


1.3.2 Benchmark Economy

Equilibrium decision rules, revenue distribution of firms and vertical relations.

Figure 8 shows the policy functions of an unattached firm. The associated values of the decision rule are as follows. The number 1 represents exit the industry, 2 stay in the industry and get a new draw of supplier (continue being unattached), 3 stay in the industry and set up a link, and 4 stay in the industry and become vertically integrated.

Figure 9. Policy function of an unattached firm.

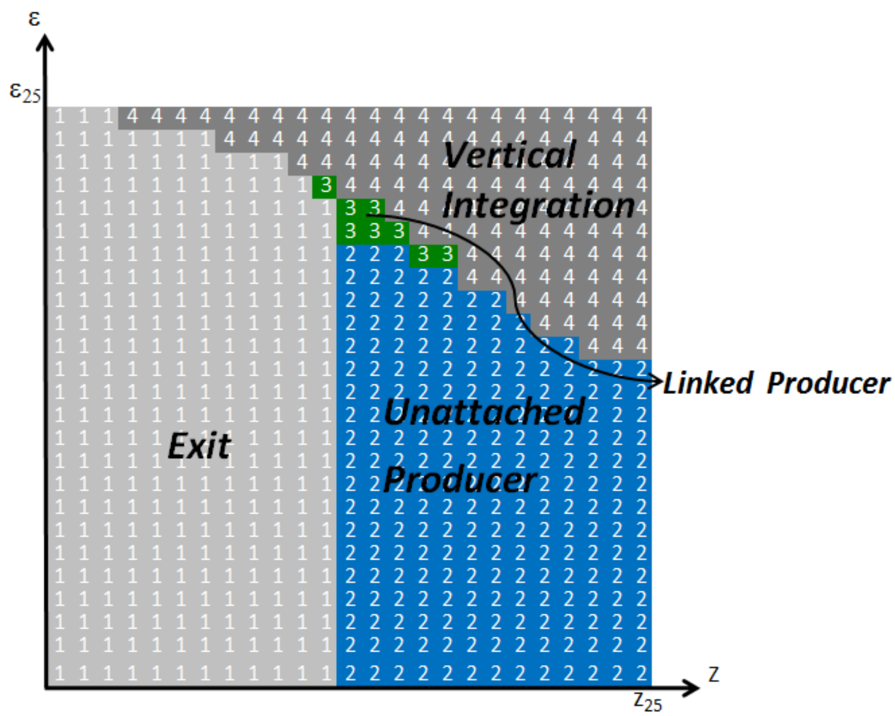


Basically, Figure 8 shows the same results derived from the theoretical section 2.3, and, in particular, the issues exposed in figure 5. The areas plotted in Figure 8 correspond to the characterization of the decision rules made in Propositions 1. Cells containing the same number define the vertical status for different firms. Besides, the least efficient firms decide to exit the industry. As it was explained in section 2, firms with pairs of productivity levels $z < z^*$ and $\varepsilon < \widehat{\varepsilon}(z)$ exit the industry (area indicated by cells containing number 1). Unattached manufacturer firms that are efficient but matched with inefficient suppliers decide to continue active and get a new draw for next period (area indicated by cells containing number 2).

The most efficient manufacturer firms (the ones with highest levels of z) decide to become vertically integrated when they are matched with efficient suppliers. There are some manufacturers with intermediate productivity levels, which have drawn an efficient supplier, and decide to keep the same supplier by setting up a link (number 3-area). The increasing differences in cost function generates the correlation of types for high productivity levels.

Figure 9 shows the decision rules of a vertically integrated firm and has the same interpretations as before. A particularly interesting point here is that the model generates vertical disintegration of plants. Moreover, identical manufacturers may differ in their vertical structure, and those that are vertically integrated can end up disintegrated or remain integrated. For example, taking a firm with high z-productivity and an intermediate upper level for ε, start decreasing the level for z and keep ε fixed (given that ε does not evolve over time). Then if its z-productivity decreases enough over time, this manufacturer will decide to disintegrate and become linked, outsourcing the input production. Furthermore, if the productivity continues to decrease, it may decide to change supplier or exit the industry.

Figure 9. Policy function of a VI firm.



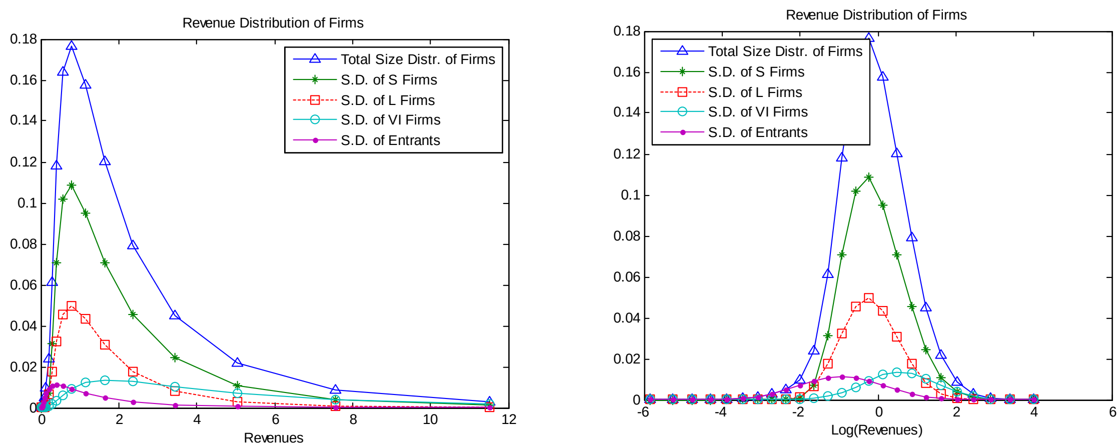
To summarize, we can see that our model induces the following behavior of firms. Vertically integrated manufacturer firms are larger and more efficient on average. Big and efficient standardized manufacturers that seek to expand though vertical integration choose suppliers that are also large and efficient as found in Hortaçsu and Syverson (2009).

In equilibrium the model generates some big manufacturers that are not vertically integrated, in line with the fact exposed in Figure 1. In Figure 10, panel A presents the equilibrium size (revenue) distribution of manufacturing plants[16]. The line with triangles represents the total size distribution of firms, while the other lines represent, for each size, the proportion of each type of firm (U, VI, L and Entrants) to the total share of firms for each particular size (this is, the area below each line

[16]Panel A in Figure 10 excludes the highest values for z so as to present a better exposition of the distributions at the lowest productivity levels. Panel B presents the whole range of the log of z.

adds up to the share of each category in the total number of plants). Panel B shows the same picture in logarithmic scale.

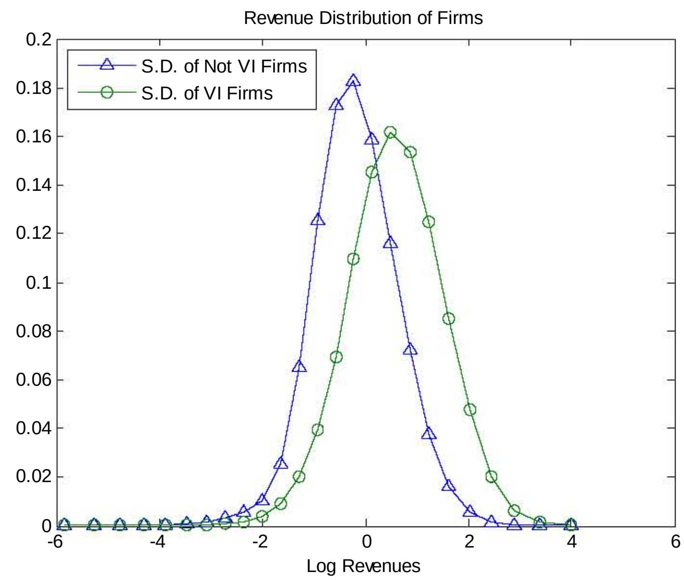
Figure 10. Size distribution of firms.



A-Revenue distribution of firms. B-Revenue dist. of firms (log. scale).

Notice that there is an overlap between these distributions: downstream firms with the same high z-productivity levels differ in their vertical structure in the steady state. The explanation for this, according to our model, is that some efficient manufacturers decide not to become vertically integrated and instead get a new draw while still looking for a more efficient supplier. The previous two graphs show that the fraction of vertically integrated plants increases with the plant size. In addition, it can also be seen that vertically integrated firms dominate (in first order stochastic dominance sense) the size distribution of not vertically integrated firms. This last fact is exposed better in Figure 11, which presents just the size distribution of vertically integrated and not vertically integrated manufacturing plants. In Figure 11 each line is the share of plants as a proportion of all plants in a particular vertical structure (the total area below each line adds up to one).

Figure 11. Size distribution of vertically integrated and not vertically integrated manufacturers.



1.3.3 How does the model work?

The model economy presented above gives rise to rich industry dynamics as manufacturers enter, exit and decide how to obtain their inputs. In this environment an industrial structure emerges as the result of optimal investment decisions that firms undertake under uncertainty. Differences across industries that affect firms' incentives to use the VI or L margins determine firm-level TFP dynamics and have an impact on profitability, survival, size distribution of firms and average productivity of an industry. In the following sections we use the model to address the questions on why supply relations vary across industries and across firms within industries, and how these relations affect size distribution of firms, turnover, mobility, welfare, aggregate output and productivity. We first study the effect that changes in the bargaining power of manufacturers, the costs to become VI, the discount factor, as well as the effects that changes in the fixed production costs have on all these relevant variables so as understand how the model works and to interpret the main mechanism of the model.

Finally, we focus on the main part of the paper, in which we address the following question: How do changes in the properties of uncertainty at firm level determine differences in the vertical structure of an industry? In order to address this question we show an experiment in which we change the persistence of the productivity shocks that manufacturers face. We show that when the productivity shocks for manufacturers are less persistent, interpreting this as higher mobility across productivity states and thus more uncertainty, manufacturers become more flexible avoiding VI and seting up links or using standardized inputs.

This experiment is important because it provides relevant empirical implications on the effect of changes in the uncertainty at firm level on firms' vertical structure, and shows how the model gives an alternative interpretation to important facts in the data. For instance, as mentioned in the introduction, the previous literature on the organization of economic activity has emphasized the role of specific investments on vertical integration, according to which in industries with high specific investments firms tend to be vertically integrated. However, there are industries with high specific investments, such as the women's dress industry (among other case studies discussed in Kranton and Minehart, 2000), in which manufacturers tend to make specific investments with specialized suppliers rather than becoming vertically integrated. The quantitative experiment shows that in industries that are also characterized by substantial uncertainty at firm level, despite the presence of specific investments, the choice between VI and link is nontrivial. It follows from the trade-off between loosing flexibility against negative shocks (under VI) and suffering the hold-up problem (sharing a fraction of profits with the supplier under links). Since industries such as the women's dress industry are also characterized by substantial uncertainty at firm level (as well as many other industries as documented by Castro, Clementi and MacDonald 2009) the interpretation of this experiment is important as it shows how the model is able to explain important facts in the data.

Bargaining power and vertical structure

In this section we analyze the effect of changes in the bargaining power of the manufacturer (Table 6). When the bargaining power of the manufacturer increases, downstream firms face a less severe hold-up problem. The average specialized input price, $p_s^L(\cdot)$, decreases from 1.39 to 1.17, which leads the manufacturers to become linked instead of vertically integrated. As it can be seen in the table, the share of vertically integrated firms decreases and the share of linked ones increases (the mass of vertically integrated and linked firms reacts in the same direction). The slight decline in the final good price from 1 to 0.98, that yields an increase in consumer surplus, together with a reduction in the average specialized input price, which generates an increase in producer surplus, yields a higher aggregate welfare. Furthermore, as the total investment increases, TFP increases.[17,18]

[17] See the appendix for definitions of total factor productivity (TFP) and revenue TFP (RTFP).

[18] Total welfare is the sum of consumer and producer surplus, which is calculated as follows:

Table 6: Changes in bargaining power

	θ		
	0.5	0.6	0.7
Price	1.00	0.99	0.98
Exit Rate	0.09	0.08	0.08
Agg. Output	100.0	100.0	100.4
TFP	100.0	102.1	103.3
Welfare	100.0	100.2	101.0
Consumer surplus	100.0	100.0	100.9
Producer surplus	100.0	100.9	105.7
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.084	0.072	0.071
$\frac{VI}{L}$	0.328	0.238	0.204

Costs of VI and L and vertical structure

Let's now focus on the specific investment cost, h. An increase in h generates a decline in the value at entry of manufacturers, and this leads to a higher final good price, lower output (thus lower consumer surplus), and a higher exit rate (Table 7). As the cost of becoming linked is higher, relative to becoming vertically integrated, the ratio VI to L rises.

Despite the increase in the exit rate, there is a decline in TFP. The lower TFP level is caused by a decrease in the TFP of suppliers. Given that small and medium sized manufacturers use links more intensively relative to VI, the increase in h has a big impact on this group of firms. In addition, as small and medium sized manufacturers are more selective in the ε they choose to invest in, this leads to lower RTFP of suppliers (from 1.8 to 1.6). In line with this reasoning, it can be seen that some firms that invested in L, now do not invest at all, and some other ones invest in VI, as shown by the increase in the percentage of median-sized firms that invest in VI from 0.052 to 0.064. As a result, even though there is higher selection, TFP

$$\begin{aligned} Welfare &= \frac{{Q^*}^{1+\eta}}{1+\eta} - pQ^* + \sum_z \sum_\varepsilon \Big[\pi_m^U(z,\varepsilon)\Phi^U(z,\varepsilon) + \pi_m^L(z,\varepsilon)\Phi^L(z,\varepsilon) \\ &\quad + \pi_m^{VI}(z,\varepsilon)\Phi^{VI}(z,\varepsilon) + \pi_s^U(z,\varepsilon)\Xi^U(z,\varepsilon) + \pi_s^L(z,\varepsilon)\Xi^L(z,\varepsilon)\Big] \end{aligned}$$

decreases, producer surplus decreases and total welfare decreases.

Table 7: Changes in specific investment and VI fixed costs.

	h			λ		
	1.3	1.4	1.5	3.15	3.25	3.35
Price	1.00	1.01	1.03	1.00	1.00	1.00
Exit rate	0.086	0.091	0.091	0.086	0.090	0.09
Agg. Output	100.0	98.9	97.9	100.0	100.0	100.0
TFP	100.0	99.1	97.6	100.0	101.4	101.4
Welfare	100.0	97.5	95.1	100.0	100.0	100.0
Consumer surplus	100.0	97.7	95.4	100.0	100.0	100.0
Producer surplus	100.0	91.9	84.2	100.0	99.9	99.6
Share of Vertically Integrated Firms						
$\frac{VI}{Total\ Firms}$	0.084	0.090	0.096	0.084	0.062	0.060
$\frac{VI}{L}$	0.328	0.427	0.589	0.328	0.251	0.242

When the additional managerial fixed cost of a vertically integrated manufacturer (λ) increases, the share of vertically integrated firms, as well as the ratio of vertically integrated to linked firms, decreases (Table 7). Furthermore, the increase in the fixed cost of a vertically integrated firm does not seem to have an effect on the value at entry of manufacturers, because the possibility to become a big vertically integrated firm is strongly discounted upon entry. Therefore, the equilibrium price remain the same as before (so does the consumer surplus), but the exit rate increases. In addition, the TFP increases a bit while producer surplus slightly decreases. Thus there is no effect on total welfare.

Complementarity and vertical structure

When T_1 increases, it increases the complementarity between manufacturer and supplier's type making the effects of cost reducing investment more important, thus the mass of firms that become vertically integrated and linked increases (Table 8). The exit rate decreases and it is cheaper to invest and thus to survive. The larger proportion of inefficient firms offsets the original decline in costs, thus the TFP decreases. Finally, total welfare increases.

Table 8: **Changes in complementarity.**

	T_1			T_2			α		
	70	75	80	-0.5	0	0.5	0.45	0.47	0.49
Price	1.02	1.00	0.97	1.06	1.00	0.838	1.00	1.00	0.99
Exit rate	0.090	0.086	0.086	0.093	0.086	0.072	0.090	0.086	0.088
Agg. Output	100.0	102.1	104.4	100.0	105.3	122.7	100.0	100.5	101.1
TFP	100.0	97.9	95.8	100.0	101.0	103.6	100.0	100.3	98.6
Welfare	100.0	104.8	109.9	100.0	112.6	158.1	100.0	101.3	102.4
Consumer surplus	100.0	104.8	110.0	100.0	112.2	157.1	100.0	101.2	102.4
Producer surplus	100.0	104.8	106.0	100.0	128.8	208.0	100.0	105.6	101.0
Share of VI									
$\frac{VI}{Total\ Firms}$	0.058	0.084	0.103	0.047	0.084	0.107	0.077	0.084	0.104
$\frac{VI}{L}$	0.235	0.328	0.431	0.294	0.328	0.233	0.331	0.328	0.480

The increase in T_2 generates an increase in the value at entry, which makes the equilibrium final good price and exit rate lower. When T_1 increases, every manufacturer increases VI and L with less efficient suppliers. In contrast, when T_2 increases it is the least efficient active manufacturers that were in the margin of setting up links and becoming vertically integrated the ones that start playing an important role in the total investment. As explained above, in Figure 5, these groups of manufacturers are more selective with respect to the supplier they choose to become vertically integrated or linked. They have to find a very efficient supplier in order to do so. Thus, an increase in T_2 generates an increase in TFP, in contrast with what happens when T_1 increases.[19]

The parameter α indicates how important is the manufacturer's type, z, relative to supplier's type, ε, in the effect of the cost reducing investment. If α increases it is less important than before, in terms of reductions in variable cost, how efficient is the supplier. Thus, when α increases it makes manufacturers less selective on the type of supplier they choose to invest in VI and L. As a result TFP decreases. In addition, the share of vertically integrated to linked manufacturers increases. Moreover, as it is easier to become more productive when linking or becoming vertically integrated (it depends less on how efficient the supplier is), the value at entry increases and

[19] The RTFP of suppliers increases from 1.6 to 2.2.

the equilibrium price decreases. The decline in final good price leads to an increase in total production and consumer surplus. Finally, total welfare increases.

Discount factor and vertical structure

With respect to a change in the discount factor, as firms value more the future they have more incentives to invest, thus the total investment in VI and L increases (the measure of vertically integrated and linked firms rise), and the share of firms using specialized inputs increases (Table 9). As the value at entry increases, the equilibrium final good price decreases and consumer surplus increases. Given that there is less selection, in equilibrium there are more inefficient firms active in the industry, and TFP decreases. Furthermore, as the decrease in TFP does not seems to have a big impact on aggregate profitability, the total producer surplus increases and, as a result, total welfare increases.

Table 9: Changes in discount factor.

	β		
	0.95	0.96	0.97
Price	1.08	1.00	0.913
Exit Rate	0.090	0.086	0.083
Agg. Output	100.0	107.5	116.3
TFP	100.0	95.5	90.9
Welfare	100.0	117.0	138.7
Consumer surplus	100.0	117.3	139.6
Producer surplus	100.0	103.0	104.4
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.078	0.084	0.091
$\frac{VI}{L}$	0.342	0.328	0.341

Fixed entry and production costs and vertical structure

When manufacturer's fixed cost of production is higher, the equilibrium price increases and consumer surplus decreases (Table 10). The exit rate increases, which generates an increase in TFP. An increase in the fixed cost of suppliers has similar effects. In both cases total welfare decreases.

The effect of changes in entry costs of manufacturers and suppliers is as follows (Table 10 and 11). When C_e^m increases, the equilibrium price increases and production, as well as consumer surplus, decreases. The increase in price generates more investments in VI, in particular by small firms (the percentage of median sized manufacturing plants that are vertically integrated increases). There is also a relative increase in the share of big firms. This explains the rise in TFP. Although there is an increase in TFP, producer surplus decreases due to the decline of entry and the total mass of firms.

Table 10: Changes in fixed costs and entry costs.

	C_f^m			C_f^s			C_e^m		
	0.35	0.40	0.45	0.35	0.40	0.45	2.5	3.0	3.5
Price	0.96	1.00	1.04	0.96	1.00	1.04	0.93	1.00	1.07
Exit rate	0.08	0.08	0.09	0.08	0.08	0.09	0.08	0.08	0.09
Agg. Output	100.0	96.77	93.28	100.0	96.77	93.28	100.0	94.06	88.40
TFP	100.0	102.13	102.41	100.0	95.04	89.45	100.0	105.32	108.18
Welfare	100.0	93.19	85.87	100.0	93.19	85.85	100.0	87.70	76.52
Consumer surplus	100.0	92.99	85.70	100.0	92.99	85.70	100.0	87.34	76.09
Producer surplus	100.0	102.81	93.71	100.0	102.81	92.74	100.0	106.46	98.93
Share of VI									
$\frac{VI}{Total\ Firms}$	0.087	0.084	0.086	0.087	0.084	0.085	0.084	0.084	0.093
$\frac{VI}{L}$	0.219	0.257	0.220	0.219	0.257	0.217	0.223	0.257	0.210

A rise in the entry cost of suppliers induces an increase in the standardized input price (from 0.42 to 0.46). Thus, there is an increase the exit rate of manufacturers and in the final good price which yields a decline in consumer surplus. The increase in the standardized input price induces an increase in VI and a decline in L.

Table 11: Changes in entry costs.

	C_e^s		
	2.5	3.0	3.5
Price	0.97	1.00	1.025
Exit Rate	0.08	0.08	0.09
Agg. Output	100.0	97.85	95.80
TFP	100.0	101.48	101.30
Welfare	100.0	95.41	90.95
Consumer surplus	100.0	95.30	90.92
Producer surplus	100.0	100.20	92.36
Share of Vertically Integrated Firms			
$\frac{VI}{Total\ Firms}$	0.084	0.084	0.093
$\frac{VI}{L}$	0.257	0.257	0.210

1.3.4 Idiosyncratic productivity shocks and vertical structure

In our framework we have three different types of manufacturer firms. First, an unattached manufacturer, which has no variable costs advantage relative to vertically integrated and linked firms. It is not subject to hold-up and has lower fixed costs relative with a vertically integrated firm. Thus it performs better when facing negative shocks.

Second, a linked firm. It uses specialized inputs and is subject to a hold-up problem. It performs better than a vertically integrated manufacturer firm when negative shocks are realized (avoid higher fixed costs and bound losses).

And third, a vertically integrated firm which has the lowest variable costs. It is not subject to hold-up. In addition, it pays higher fixed costs and requires higher investment costs ($h+P_{VI}$, which in equilibrium is much higher than h), then perform worst (have larger losses) when facing negative shocks.

In this section we want to address the following question: what is the implication of making the evolution of the manufacturers productivity shocks less persistent? In table 12 we present the comparative statics results. It shows the effect of decreasing the persistence of shocks, ρ, on the vertical relation of the industry. By comparing the first column with the other ones, it can be seen that the share of vertically integrated manufacturers to linked ones decreases, as well as the share of vertically integrated manufacturers, while the mass of vertically integrated firms decreases and the measure of linked ones increases. Moreover, the share of firms that invest in using specialized inputs, $(VI + L)/Total\ Firms$, increases.

Because of cost reducing investment through VI are less attractive when there is a decline in the persistence, manufacturers value at entry decreases. Hence the equilibrium price increases. As a result, the equilibrium output decreases and consumer surplus is lower. In addition, the increase in the final good price generates a lower exit rate. Despite the lower selection, there is an increase in producer surplus and total factor productivity (TFP) due to the fact that efficient manufacturers that invest in the use of specialized inputs become more selective about suppliers' type. In other words, in order to invest in VI or L, manufacturers wait more until they get matched with a better supplier. Thus suppliers' productivity increases.[20] Finally, as the decline in consumer surplus is bigger than the increase in producer surplus, the total welfare decreases.

Table 12: Changes in persistence and variance of shocks.

	ρ			σ^2_ξ		
	0.93	0.92	0.91	0.13	0.15	0.17
Price	1.00	1.08	1.11	0.90	1.0	0.99
Exit rate	0.09	0.06	0.06	0.05	0.086	0.092
Agg. Output	100.0	93.6	91.4	100.0	91.3	92.3
TFP	100.0	104.2	108.4	100.0	98.7	93.8
Welfare	100.0	86.8	82.9	100.0	81.2	82.5
Consumer surplus	100.0	86.5	81.9	100.0	81.9	83.9
Producer surplus	100.0	112.6	132.8	100.0	60.0	39.7
Share vertically integrated Firms						
$\frac{VI}{Total\ Firms}$	0.084	0.062	0.059	0.130	0.084	0.041
$\frac{VI}{L}$	0.328	0.206	0.152	0.308	0.328	0.385

If ρ is high, firms anticipate that high shocks today will be around for long time. Thus, by becoming vertically integrated, they strongly discount the realization of a low shock (while paying high fixed costs). Therefore, many firms decide to become vertically integrated.

In contrast, if ρ is low, there is higher mobility across productivity states and the expected duration of being in a high idiosyncratic efficiency level is lower. There is

[20] Revenue TFP of suppliers increases significantly, from 1.8 to 2.1, while the RTFP of manufacturers does not change.

a higher possibility of having a low shock relatively soon, incurring high losses (due to high fixed production costs) or not recovering the investment cost $(h + P_{VI})$. As a result, manufacturers become more flexible, which is reflected by a lower VI to L ratio and a decrease in the share of vertically integrated firms.

To summarize, as found in Kranton and Minehart (2000), our result indicates that the properties of the idiosyncratic risk at firm level plays an important role in determining the vertical structure of firms. The choice of manufacturers between VI and link is nontrivial. It follows from the trade-off between loosing flexibility against negative shocks and sharing a fraction of profits with the supplier.

As the variance σ_{ξ}^2 increases, given that the per period profit is concave in z, the value at entry is lower. Hence the equilibrium price increases and consumer surplus shows a large decline. A higher dispersion in productivity shocks implies that there are entrants with efficiency levels within a wider range of values. The most inefficient ones exit while the most efficient ones survive (each one of which contributes more to total production than before). Thus, there are two forces that diminishes the total number of firms. First, the higher equilibrium prices generates a decline in demand, and therefore there is less space for production units in the market. And second, there are bigger production units that satisfy the lower quantity demanded. What is interesting here is that, even though there is a reallocation of resources from small to medium and big firms (looking at the size distribution of firms, there is an increase in the share of big firms and a decline in the share of small ones), which increases the RTFP of manufacturers, the big decline in total investment (the share, as well as the mass, of firms that become vertically integrated and linked decreases) generates lower supplier's RTFP. As a result, the total RTFP (and TFP) decreases. In line with this, producer surplus is lower, hence total welfare decreases.

1.4 Conclusion

This paper proposes a dynamic entry and exit model of an industry with vertical structure decisions and specific investments. In the model, the industrial vertical structure is the result of optimal investment decisions that firms make under uncertainty. The model does well in replicating new facts on vertical structures documented in Hortaçsu and Syverson (2009) and Kranton and Minehart (2000). Our results indicate that differences in vertical structures across industries, and across firms within industries, are the result of differences in the properties of the stochastic process governing the uncertainty at firm level, in specific investment costs, in bargaining power of manufacturers and suppliers, and in complementarity of manufacturers' and suppliers' productivity.

1.5 Appendix

1.5.1 Stationary Equilibrium

Because there is a continuum of firms that are subject to idiosyncratic shocks, there is a cross sectional distribution of firms over the states (z, ε) and over different vertical structures. We call Φ^U the stationary distribution of downstream unattached firms, and Φ^{VI}, Φ^L, Ξ^U and Ξ^L the stationary distribution of vertically integrated manufacturers, linked manufacturers, unattached suppliers and specialized suppliers, respectively. Let′s define $D(p)$ as the aggregate demand, that is continuous and strictly decreasing. Then, the stationary equilibrium is standard:

A stationary equilibrium in this model is a list of value functions for manufacturers and suppliers ($V^U(z,\varepsilon)$, $V^L(z,\varepsilon)$, $V^{VI}(z,\varepsilon)$, $W^U(z,\varepsilon)$, $W^L(z,\varepsilon)$, $V_e^m(p,p_s)$, $W_e^s(p,p_s)$), policy functions ($a_U(z,\varepsilon)$, $x_U'(z,\varepsilon)$, $a_L(z,\varepsilon)$, $a_{VI}(z,\varepsilon)$, $x_{VI}'(z,\varepsilon)$), prices p and p_s and price functions $p_s^L(z,\varepsilon)$ and $P_{VI}(z,\varepsilon)$, invariant measures for downstream standardized firms Φ^U, vertically integrated firms Φ^{VI} and linked firms Φ^L and invariant measures for upstream unattached firms Ξ^U and upstream linked firms Ξ^L, an invariant density $J^m(z)$, a mass of downstream and upstream entrants μ^m and μ^s, a threshold z^* and a threshold function $\widehat{\varepsilon}(z)$, given the aggregate demand function for final goods $D(p)$ such that:

i) Input prices $p_s^L(z,\varepsilon)$ and acquisition prices $p_{VI}(z,\varepsilon)$ are given by NBS

ii) Given p, p_s, $p_s^L(z,\varepsilon)$ and $P_{VI}(z,\varepsilon)$, policy functions $a_U(z,\varepsilon)$, $a_{VI}(z,\varepsilon)$ and $a_L(z,\varepsilon)$ solve the static input decisions

iii) Given p, p_s, $p_s^L(z,\varepsilon)$ and $P_{VI}(z,\varepsilon)$, policy functions $x_U'(z,\varepsilon)$ and $x_{VI}'(z,\varepsilon)$ solve the dynamic decisions of firms

iv) Free entry conditions are satisfied for manufacturers

$$C_e^m = V_e^m(p;p_s) = \sum_{\varepsilon}\sum_{z} V^U(z,\varepsilon) g^m(z) g^s(\varepsilon), \tag{1.15}$$

and for suppliers

$$C_e^s = W_e^s(p_s;p) = \sum_{\varepsilon}\sum_{z} W^U(z,\varepsilon) J^m(z) g^s(\varepsilon). \tag{1.16}$$

v) Market clearing conditions are satisfied in the market for final goods $D(p) =$

$S(p)$ and in the market for standardized inputs $D^s(p_s) = S^s(p_s)$ where

$$S(p) = \sum_z \sum_\varepsilon z a_U(z,\varepsilon)\Phi^U(z,\varepsilon) + \sum_z \sum_\varepsilon z a_{VI}(z,\varepsilon)\Phi^{VI}(z,\varepsilon) + \sum_z \sum_\varepsilon z a_L(z,\varepsilon)\Phi^L(z,\varepsilon). \tag{1.17}$$

$vi)$ Laws of motion of states are consistent with individual decisions (stationary measures Φ^U,Φ^{VI},Φ^L,Ξ^U and Ξ^L are fixed points). As mentioned before the heterogeneity of a market firm is described by $\Phi^U(B)$ measure on $(S,\mathcal{B})$, where $S = Z \times E$ and B_s = all possible subsets of S, and $B\epsilon\mathcal{B}_s$. Then we have the following fixed point of the form $\Phi^U = T(\Phi^U,\mu^m)$:

$$\begin{aligned}\Phi^U(B) = {} & \underbrace{\sum_z^{s_n}\sum_\varepsilon^{s_n} \underbrace{\Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right)}_{\text{Element of the Markov chain}} \underbrace{I_{(x'_U(z,\varepsilon)=U)}}_{\text{Indicator function from policy functions}} \Phi^U(z,\varepsilon)}_{\text{Incumbent who survive}} \\ & + \underbrace{\sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_{VI}(z,\varepsilon)=U)}\Phi^{VI}(z,\varepsilon)}_{\text{Vertically Integrated Incumbent who survive}} \\ & + \underbrace{\sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_L(z,\varepsilon)=U)}\Phi^L(z,\varepsilon)}_{\text{Linked Incumbent who survive}}\ \forall B\epsilon\mathcal{B}_s. \\ & \underbrace{\sum_z^{s_n}\sum_\varepsilon^{s_n} \underbrace{\Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right)}_{\text{Element of the Markov chain}} \underbrace{I_{(x'_U(z,\varepsilon)=U)}}_{\text{Indicator function from policy functions}} \mu^m g^m(z) g^s(\varepsilon))}_{\text{Entrants}}\end{aligned} \tag{1.18}$$

In a similar way, the heterogeneity of incumbent downstream firms that are vertically integrated and linked is described by $\Phi^{VI}(B)$ measure on $(S,\mathcal{B})$ and $\Phi^L(B)$ measure on $(S,\mathcal{B})$. Then we have $\Phi^{VI} = T^{VI}(\Phi^{VI})$ and $\Phi^L = T^L(\Phi^L)$:

$$\begin{aligned}\Phi^{VI}(B) = {} & \sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_U(z,\varepsilon)=VI)}\Phi^U(z,\varepsilon) \\ & + \sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_{VI}(z,\varepsilon)=VI)}\Phi^{VI}(z,\varepsilon) \\ & + \sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_L(z,\varepsilon)=VI)}\Phi^L(z,\varepsilon)\ \forall B\epsilon\mathcal{B}_s.\end{aligned} \tag{1.19}$$

And finally, we have the following fixed point for the measures of linked firms

$$\begin{aligned}\Phi^L(B) = \ & \sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_U(z,\varepsilon)=L)}\Phi^U(z,\varepsilon) \\ & +\sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_{VI}(z,\varepsilon)=L)}\Phi^{VI}(z,\varepsilon) \\ & +\sum_z^{s_n}\sum_\varepsilon^{s_n} \Pr\left((z',\varepsilon')\,\epsilon B|z,\varepsilon\right) I_{(x'_L(z,\varepsilon)=L)}\Phi^L(z,\varepsilon)\ \forall B\epsilon\mathcal{B}_s.\end{aligned} \tag{1.20}$$

vii) The mass of suppliers, μ^s, equal the mass of unattached and linked manufacturers

$$\Xi^U + \Xi^L = \sum_z\sum_\varepsilon \Phi^U(z,\varepsilon) + \sum_z\sum_\varepsilon \Phi^L(z,\varepsilon)$$

1.5.2 Solution Method

The algorithm to compute the equilibrium is as follows:

1) Given initial guesses for the price of the final good, p^0, and for the standardized input price, p_s^0, compute the price for the specialized input, $p_s^{L0}(z,\varepsilon)$, by NBS over current profits, that is, taking

$$p_s^{L0} = p_s - (1-\theta)c(z,\varepsilon),$$

as the solution of expression (8), and take P_{VI} as $\frac{p_s - C_f^m}{1-\beta} + h$. Take these prices as the initial guesses for p_s^{L0} and P_{VI}^0

2) Take an initial guess for the density of productivity of manufacturers looking for a standardized suppliers $J_0^m(z)$,

3) Obtain policy functions $a_U(\cdot)$, $x'_U(\cdot)$, $a_L(\cdot)$, $a_{VI}(\cdot), x'_{VI}(\cdot)$ and value functions $, V^U(\cdot)$, $V^{VI}(\cdot)$, $V^L(\cdot)$, $W^U(\cdot)$ and $W^L(\cdot)$ (equations $1, 2, 4, 5,$ and 6).

4) Compute the price for the specialized input, $p_s^L(z,\varepsilon)$ by NBS taking into account the continuation values (equation 8) and $P_{VI}(z,\varepsilon) = \beta E_{z',\varepsilon'}W^U(z',\varepsilon')$.

5) Compare $p_s^L(z,\epsilon)$ and $P_{VI}(z,\varepsilon)$ with previous guesses $p_s^{L0}(z,\epsilon)$ and $P_{VI}^0(z,\varepsilon)$.

   i) If they are close⇒guess a new specialized input price, taking:

$$\begin{aligned} p_s^{L0}(z,\epsilon) &= p_s^{L0}(z,\varepsilon) + \Lambda(p_s^L(z,\varepsilon) - p_s^{L0}(z,\varepsilon)), \text{ and} \\ P_{VI}^0(z,\varepsilon) &= P_{VI}^0(z,\varepsilon) + \Lambda(P_{VI}(z,\varepsilon) - P_{VI}^0(z,\varepsilon)),\end{aligned}$$

   where Λ is a convergence tolerance parameter, and repeat from point (3).

ii) If they are close⇒compute for each price $p_s^L(z,\varepsilon)$ and $p_{VI}(z,\varepsilon)$ the gains from trade for manufacturers and suppliers that trade inputs:

  * If for some (z,ε) gains from trade are negative⇒use an indicator so that under these prices the manufacturer decides not to negotiate, and repeat from point (3) using these new prices.
  * If for every (z,ε) gains from trade are positive⇒stop and go to next point.

6) Use the computed decision rules and the transition matrix to compute the invariant density of productivity of manufacturers looking for unattached suppliers $J^m(z)$, and compare it with $J_0^m(z)$:

   i) If they are not close⇒guess a new one ($J_0^m(z) = J^m(z)$) and repeat from point (2) until they get close.

   ii) If they are close⇒stop and go to next point .

7) Compute $V_e^m(p_s, p)$ and $W_e^s(p_s, p)$ and given the entry costs C_e^m and C_e^s verify if free entry conditions (equations 10 and 11) hold:

   i) If they do not hold:

      * If $V_e^m(p, p_s) < C_e^m$ and/or $W_e^s(p_s) < C_e^s \Rightarrow$ guess a new higher prices, p and p_s by bisection and repeat from point (1).
      * If $V_e^m(p, p_s) > C_e^m$ and/or $W_e^s(p_s) > C_e^s \Rightarrow$ guess a new lower prices, p and p_s by bisection and repeat from point (1).

   ii) If $V_e^m(p, p_s) \approx C_e^m$ and $W_e^s(p_s) \approx C_e^s$ ⇒stop and go to next point.

8) Use the computed decision rules and the transition matrix to compute the fixed points of the distribution of manufacturer firm sizes when the mass of firms is one ($\mu^m = 1$). Thus, we have the fixed points $\widehat{\Phi}^U$, $\widehat{\Phi}^{VI}$ and $\widehat{\Phi}^L$.

9) Use the linear homogeneity of the $T's$ operators (defined in point vi of the stationary equilibrium definition) in μ^m to obtain the equilibrium value for μ^m that satisfies the market clearing condition for the final good: $D(p) = S(p, \mu^m)$.

1.5.3 Physical and revenue TFP

In this section I describe how the physical and revenue total factor productivity is calculated. We denote physical and revenue total factor productivity as TFP and

RTFP, respectively. The expression for the revenue TFP is as follows:

$$\begin{aligned} RTFP = \quad & \sum_z \sum_\varepsilon a_U(z,\varepsilon) \tfrac{pz}{p_s + C_f^m} \widetilde{\Phi}^U(z,\varepsilon) + \sum_z \sum_\varepsilon a_L(z,\varepsilon) \tfrac{pz + c(z,\varepsilon)}{p_s^L(z,\varepsilon) + C_f^m} \widetilde{\Phi}^L(z,\varepsilon) \\ & + \sum_z \sum_\varepsilon a_{VI}(z,\varepsilon) \tfrac{pz + c(z,\varepsilon)}{C_f^m + C_f^{VI}} \widetilde{\Phi}^{VI}(z,\varepsilon) + \sum_z \sum_\varepsilon \tfrac{p_s}{C_f^s} \widetilde{\Xi}^U(z,\varepsilon) \\ & + \sum_z \sum_\varepsilon \tfrac{p_s^L(z,\varepsilon)}{C_f^s} \widetilde{\Xi}^L(z,\varepsilon), \end{aligned}$$

where the first term represents the weighted average (the weight is the share of unattached manufacturers in each state, $\widetilde{\Phi}^U(z,\varepsilon)$) of the ratio of standardized manufacturer's revenues, pz, to their total production cost, $p_s + C_f^m$.

The second and third terms are the weighted average of the ratio of linked and vertically integrated manufacturer's revenues to their corresponding total production costs. In these cases $\widetilde{\Phi}^L(z,\varepsilon)$ and $\widetilde{\Phi}^{VI}(z,\varepsilon)$ are the share of linked and vertically integrated manufacturers, respectively. In contrast with the first term, in the numerator it appears the variable cost advantage of specific investments, $c(z,\varepsilon)$. The other difference is in the denominator, where it appears as cost of the linked firms the bargained input price $p_s^L(z,\varepsilon)$; and for vertically integrated firms the additional fixed cost C_f^{VI}.

The last two terms correspond to the RTFP of suppliers. There, $\widetilde{\Xi}^U(z,\varepsilon)$ and $\widetilde{\Xi}^L(z,\varepsilon)$ are the share of unattached and linked suppliers, respectively. The fourth term is the RTFP of a standardized supplier, which is the ratio of revenue, p_s, to total cost, C_f^s. For specialized suppliers, the RTFP is similar, but their revenue is $p_s^L(z,\varepsilon)$.

The expression for TFP is as follows

$$\begin{aligned} TFP = \quad & \sum_z \sum_\varepsilon a_U(z,\varepsilon) \tfrac{z}{1 + \frac{C_f^m}{p}} \widetilde{\Phi}^U(z,\varepsilon) + \sum_z \sum_\varepsilon a_L(z,\varepsilon) \tfrac{z + \frac{c(z,\varepsilon)}{p}}{1 + \frac{C_f^m}{p}} \widetilde{\Phi}^L(z,\varepsilon) \\ & + \sum_z \sum_\varepsilon a_{VI}(z,\varepsilon) \tfrac{z + \frac{c(z,\varepsilon)}{p}}{\frac{C_f^m + C_f^{VI}}{p}} \widetilde{\Phi}^{VI}(z,\varepsilon) + \sum_z \sum_\varepsilon \tfrac{1}{\frac{C_f^s}{p}} \widetilde{\Xi}^U(z,\varepsilon) \\ & + \sum_z \sum_\varepsilon \tfrac{1}{\frac{C_f^s}{p}} \widetilde{\Xi}^L(z,\varepsilon), \end{aligned}$$

in which the difference with the definition for RTFP is the following. For manufacturers, every term reflects the ratio of units produced by each firm to the units of all inputs they use in production. Unattached and linked manufacturers use one unit of input to produce and $\frac{C_f^m}{p}$ fixed units of physical resources to produce z and $z+\frac{c(z,\varepsilon)}{p}$ units of final goods, respectively. Every vertically integrated firm produces $z+\frac{c(z,\varepsilon)}{p}$ units of final goods and uses $\frac{C_f^m+C_f^{VI}}{p}$ fixed units of physical resources to produce. The logic is the same for suppliers.

Chapter 2

Dual Employment Protection Legislation and the Size Distribution of Firms *(joint with Andrés Erosa)*

2.1 Introduction

In this paper, we develop a new theory in which dual employment protection legislation (DEPL) have non trivial effects on the size distribution of firms and aggregate total factor productivity (TFP) by distorting firm selection and the resource allocation among firms. Our theory is motivated by three observations: 1) there exist evidence showing that differences in the size distribution of firms across countries is important to account for differences in aggregate productivity across countries; 2) there is evidence documenting that strict DEPL tended to be associated with a higher share of temporary employment across countries and lower productivity; and 3) we present data indicating that countries with particularly high fraction of temporary employment have a relatively large share of employment allocated in small firms.

We develop a theory of firm dynamics with search frictions and asymmetric firing costs of permanent and temporary workers in order to study the effect of dual employment protection legislation on the size distribution of firms and aggregate TFP. In the model economy, from a firm's perspective, DEPL puts in place two different workers. Temporary workers that remain matched with a firm for a short period of time (they have an exogenous separation rate equal to one) and are dismissed at zero firing cost; and permanent workers that remain matched with a firm until it decides to fire them (they have an exogenous separation rate equal to zero), but have high firing costs. In this framework firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. From one hand, firms that expect their productivity to growth and therefore survive and last longer in the market have incentives to hire more permanent workers. From the other hand, firms with bad prospects on the evolution of their productivity have incentives to hire more temporary workers since they keep reducing production and employment over time and exit the industry relatively soon. In this context, an increase in the firing costs of permanent workers distorts the optimal employment composition that firms choose over their life-cycle penalizing relatively more to firms with high productivity growth. Hence, an increase in firing costs of permanent workers reduces the mass of businesses hiring permanent workers. In addition there are general equilibrium effects that reinforces this result. First, on the intensive margin, an increase in the firing costs of permanent workers reduces profits of high productivity growth firms inducing to less vacancy posting and reducing the labor market tightness. In turn, this increases the probability that a firm matches with a worker making low productivity firms to expand. Second, an increase in the firing costs of permanent workers subsidizes firms with low productivity growth by reducing the costs of filling up temporary jobs which distorts the exit decision of firms. As a result, low productivity growth firms last longer in the market and the age of shutdown of high productivity growth firms decreases. Adding up, larger firing costs for permanent contracts shifts employment from high productivity growth firms, which contract and last shorter in the market, to firms with low productivity growth, which expand and last longer in the market.

Our paper is closely related to the large literature on labor market regulations and firm dynamics.[1,2] Most papers studying the effects of separation taxes have focused on the analysis of unemployment and worker turnover. However, our contribution is to show that DEPL has nontrivial effects on the size distribution of firms and TFP. In addition, the literature has emphasized positive effects of temporary contracts since they provide firms considerable flexibility in the hiring and firing process.[3]

[1] Regarding the literature focusing on differences in labor market regulations and their outcomes across countries, see for example Blanchard and Summers (1986), Blanchard and Wolfers (2000), Layard, Nickell, and Jackman (1991), Ljungqvist and Sargent (1998), Machin and Manning (1999), and Freeman (2007).

[2] Regarding the literature focusing on separation taxes, see for example Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993), Millard and Mortensen (1997), Güell (2000), among others. Regarding the extensive literature studying the effects of temporary contracts, see for example Bentolila and Saint Paul (1992), Cabrales and Hopenhayn (1997), Blanchard and Landier (2002), Nagypal (2002), Aguiregabiria and Alonso-Borrego (2004), Alonso-Borrego et al. (2005), Veracierto (2007), and Alvarez and Veracierto (2012).

[3] For instance, Aguirregabiria and Alonso-Borrego (2009) develops a dynamic structural models of labor demand to analyze longitudinal Spanish firm-level data during the period 1982-1993 (before and after the reform), and their results indicate an important positive effects on total employment and job turnover, and small effects on labor productivity and the value of firms. Within the papers studying the effects of temporary contracts the paper by Alonso-Borrego et al. (2005) and Alvarez and Veracierto (2012) are probably the most closely related to our paper since, in contrast with the other papers, they consider firm

However, another contribution of our paper is to show that there are also negative effects of higher flexibility. Our model shows a mechanism through which higher flexibility in the hiring and firing process distorts the equilibrium selection of firms and the allocation of resources among firms.

Of course, there is an empirical literature analyzing employment protection legislation and showing that DEPL is widely used across countries and it has potential effects on employment (and unemployment), worker turnover and productivity.[4,5] Regarding employment protection legislation Autor et al. (2007) and Bassanini et al. (2008) provide empirical evidence showing that strict employment protection legislation has a depressing impact on productivity because it reduces the level of risk that firms are ready to endure in experimenting with new technologies or because there is less threat of layoff in response to poor work performance. With regards to the effect of DEPL on productivity, Boeri and Garibaldi (2007), Sanchez and Toharia (2000), and Alonso-Borrego (2010) find a negative relationship between the share of temporary workers and firms' labour productivity. Dolado and Stucchi (2008) suggest that workers on temporary contracts may be motivated to exert low effort levels because of the high probability of being fired at the end of their contracts. They attribute one-third of the fall of TFP in Spanish manufacturing firms during the period 2001–2005 to the disincentive effects of the low conversion rates on temporary workers' effort.[6] Our paper provides a new mechanism to interpret the negative effect of DEPL on TFP documented in these papers.

There is an important recent macroeconomic literature analyzing the sources of resource misallocation among production units. For instance, Erosa and Hidalgo (2008) and Buera, Kaboski and Shin (2011) focus on financial market imperfections as a source of misallocation. Hsieh and Klenow (2009) study the impact of misallocation across establishments in explaining productivity in manufacturing in

dynamics in a general equilibrium model. However, Alonso-Borrego et al. (2005) find that temporary contracts increase productivity. In the paper by Alvarez and Veracierto (2012) the main argument is that the presence of temporary contracts provides an employment buffer that firms can use to adjust to their idiosyncratic shocks without having to incur firing costs.

[4] Belgium, Denmark, Germany, Greece, Italy, Netherlands, Portugal, Spain and Sweden have introduced or intensified the use of temporary contracts since the mid 80's (European Commision, 2010).

[5] Some papers use longitudinal data of countries exploiting the differences in severance pay across countries. For instance, Lazear (1990), Addison and Grosso (1996), Burgess, Knetter and Michelacci (2000), Heckman and Pagés (2004), Abraham and Houseman (1993, 1994), Bover, García-Perea and Portugal (2000), among others. While a second line of research has exploited data before and after specific reforms in the labor market using a differences-in-differences approach. See for example Kugler (2004), Hunt (2000), and Bentolila and Saint-Paul (1992), among others. For the effects on employment and worker turnover see, the surveys by Dolado, García-Serrano and Jimeno (2002) and by Bentolilla, Dolado and Jimeno (2008) on the Spanish experience and the extensive literature cited therein.

[6] Dolado, Ortigueira and Stucchi (2012) propose a model in which both temporary workers' effort and firms' temp-to-perm conversion rates decrease when the gap in firing costs between permanent and temporary workers increases. In addition, they test the implications of the model using as natural experiments some labour market reforms entailing substantial changes in firing costs gap and they find that reforms leading to a lower gap enhanced conversion rates and increased firms' TFP.

China and India. Furthermore, they recover the underlying distortions from observed allocations and, as well as Bertelsman, Haltiwanger and Scarpetta (2008), follow Restuccia and Rogerson (2008) and model distortions as firm or plant-specific. Moreover, a key insight in Restuccia and Rogerson (2008) is that idiosyncratic distortions are more important (have the potential to do much more damage) when they are positively correlated with firm productivity (establishments with low TFP receive a subsidy and establishments with high TFP are taxed). In our model, firing costs to permanent workers act as a subsidy to small firms (which are intensive in the use of temporary workers) and a tax to big firms (which are intensive in the use of permanent workers). In this sense, our paper is related to Restuccia and Rogerson (2008). In addition, Guner, Ventura and Xu (2008) consider policies that directly target the size of the establishment (size-dependent policies) such as a tax on establishments with more than given number of employees. When a general configuration of these policies are restricted to achieve a given reduction in average establishment size, they find a substantial reduction in aggregate output per worker. In line with this idea, our paper shows that DEPL plays similar role as a size-dependent policy, penalizing more to firms that use permanent employees (big firms).

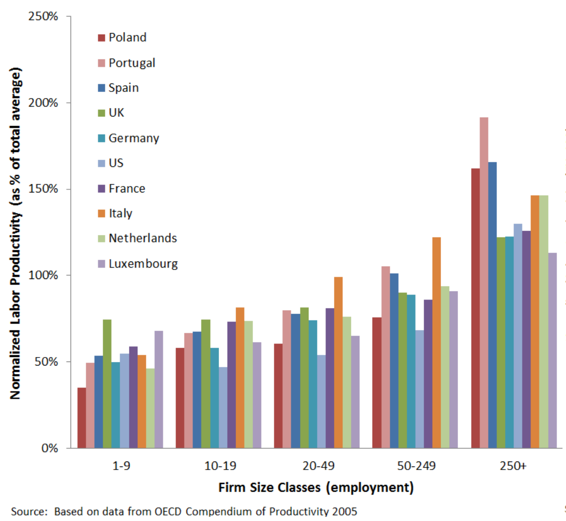
To summarize, we provide evidence showing that countries with dual employment protection legislation that incentives or extend the use of temporary contracts have relatively more employment concentrated in small firms and thus have lower productivity. Motivated by the evidence, we next develop a theoretical model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers and characterize the equilibrium labor composition that firms with different productivity growth rate choose over their life cycle. In this framework we analyze the impact of higher firing costs of permanent workers in the distribution of firms' size and productivity.

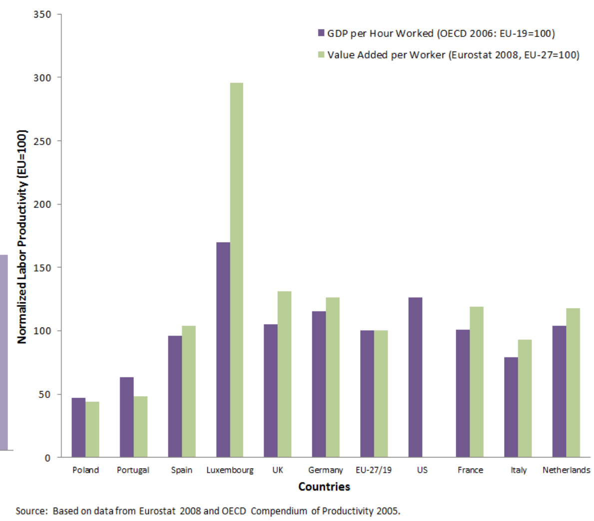
2.1.1 Empirical evidence

In order to motivate our theory, in this section we bring along evidence showing that it is important to analyze the size distribution of firms to understand differences in productivity across countries. We also show that, across countries, DEPL tends to be associated with a higher share of temporary employment, and that countries with particularly high fraction of temporary employment have a relatively large share of small firms as well as a high share of employment allocated in small firms and lower aggregate productivity.

Taking data from the OECD, Compendium of Productivity (2005), in the panel to the left of Figure 1 we present the normalized firms's labor productivity by size-classes (employment bins) in the manufacturing sector for US and many European countries.[7] The panel to the right of Figure 1 presents the aggregate labor productivity for the same group of countries.[8] Despite countries differ significantly in their aggregate labor productivity, across countries firms display similar labor productivity levels within the same size-classes. The biggest enterprise size class has the highest productivity. This is a common pattern in Europe since for the majority of countries, about 75 % (taking into account other European economies not presented here), productivity increases monotonically with size class.

Figure 1: Normalized Productivity (manufacturing sector), 2005.





How can significant differences in aggregate labor productivity across countries be reconciled with the fact that firms display similar labor productivity levels within the same size-classes across countries? The data exposed in Figure 2 and Table 1 suggests that one possible reason is that the distribution of firm sizes is different across counties. In Figure 2, panel A presents the cumulative fraction of firms for different employment levels comparing three economies, US, UK and Spain; and Table 1 shows the size distribution of firms and the labor productivity. It is clear that in Europe there is a higher concentration of firms in low size levels, measured by employment, than in US. While 0.12% of firms in Spain and 0.16% of firms in UK have more than 500 employees, in US a fraction of 0.31% of firms belong to that employment size-class. Furthermore, the fraction of firms with 100 to 499 and 20 to 99 employees is 0.75% and 4.67% in Spain, 1.31% and 7.44% in UK, and 1.52% and 8.88% in US, respectively. In addition, Spain and UK have a higher fraction of

[7] The normalised labour productivity is calculated as value added per worker in a given size class as a percentage of the average labour productivity across all size classes (see OECD, Compendium of Productivity 2005).

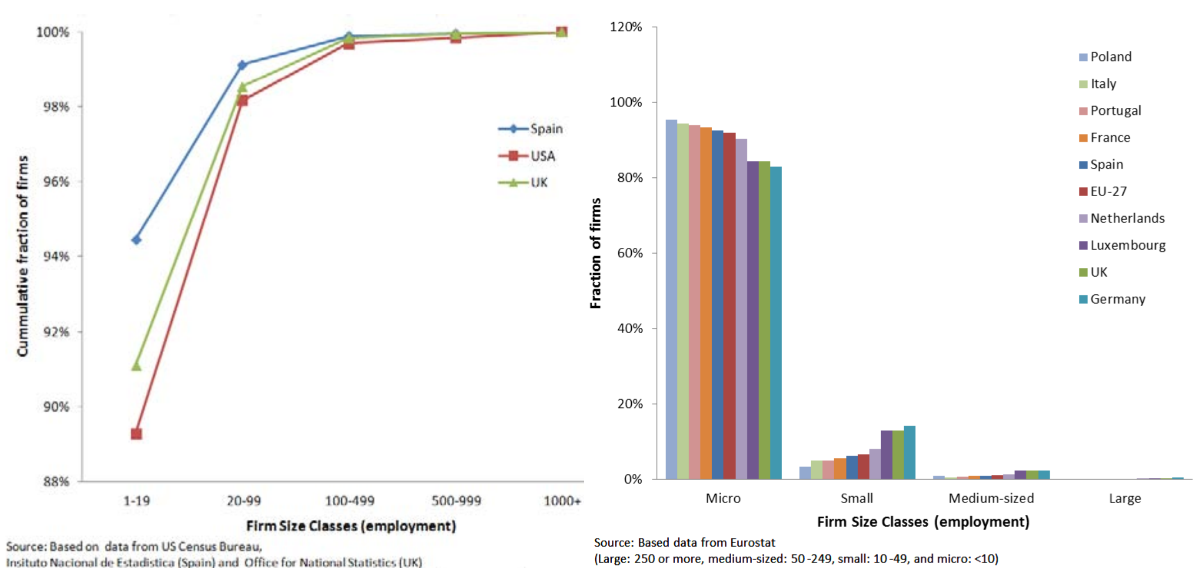
[8] The Labor productivity in the panel to the right of Figure 1 is Gross Domestic Product at constant prices and using PPP's, divided by either total employment or total hours worked.

firms in the lowest employment level, 94.46% and 91.10% in the 1 to 9 workers bin, respectively, than US (with 89.29%).

Panel B of Figure 2 presents a similar comparison among many other European countries. For instance, France, Italy and Spain have a higher fraction of micro and small firms and a lower fraction of medium and large firms than the average among European Countries. In contrast, Germany, UK and Netherlands have a lower fraction of micro and small firms and a higher fraction of medium and large firms than the average among European Countries.

Table 2 shows that the same pattern occurs when comparing the fraction of total employment concentrated in different firm size classes. For instance, in Spain small firms concentrate a bigger fraction of total employment than the average small firms in the EU. In Spain the joint employment of micro, small and medium firms represents 82.2%, versus 73.9% in the EU. In contrast, small firms in the UK employ a lower fraction of total employment with respect to the average small firms in the EU (62.87% versus 73.9%). The opposite evidence is found when we look at the fraction of employment at big firms. In Spain and the UK big firms concentrates 17.8% and 37.2% of total employment, respectively.

Figure 2: Fraction of firm by size-classes (manufacturing sector), 2008.



Panel A Panel B

Table 1: Firms size distribution and productivity.

Country	Employment-Size Classes (% of firms)				Productivity (EU-27=1)
	Micro	Small	Medium	Large	Value added per worker
Poland	95.5	3.3	1.0	0.2	0.44
Italy	94.3	5.1	0.5	0.1	0.93
Portugal	94.0	5.1	0.7	0.1	0.48
France	93.3	5.6	0.9	0.2	1.19
Spain	93.1	6.0	0.8	0.1	1.04
Netherlands	90.4	8.0	1.4	0.3	1.18
Luxembourg	84.0	12.9	2.4	0.3	2.96
UK	89.3	8.8	2.5	0.4	1.31
Germany	83.0	14.1	2.4	0.5	1.26
EU-27	92.0	6.7	1.1	0.2	1.00

Source: Taken from Eurostat 2009

Large: 250 or more, medium-sized: 50-249, small: 10-49, and micro: <10

Table 2: Employment by firms sizes.

Country	Employment-Size Classes (% of firms)			
	Micro	Small	Medium	Large
Poland	.	.	.	.
Italy	61.3	15.8	9.1	13.9
Portugal	57.7	17.1	11.9	13.3
France	35.9	17.7	13.8	32.6
Spain	49.6	20.6	12.0	17.8
Netherlands	41.1	17.8	13.9	27.2
Luxembourg	24.7	22.3	21.9	21.1
UK	35.3	14.8	12.7	37.2
Germany	32.6	18.2	16.2	33.0
EU-27	43.8	16.5	13.5	26.1

Source: Taken from Eurostat 2009

Large: 250 or more, medium-sized: 50-249, small: 10-49, and micro: <10

All this evidence suggests that is important to study the determinants of the size distribution of firms to interpret the differences in relative aggregate productivity. In line with this fact, there are other papers showing that in order to understand the cross country differences of economies' performance in many dimensions it is relevant to study the size distribution of firms. For instance, Navaretti, et al (2011), in the second EFIGE policy report, find that European countries perform very differently in terms of their trade competitiveness (exports and global production strategies) because the within-country distribution of firms characteristics (size, innovative capacity and productivity) differ significantly across these economies.[9]

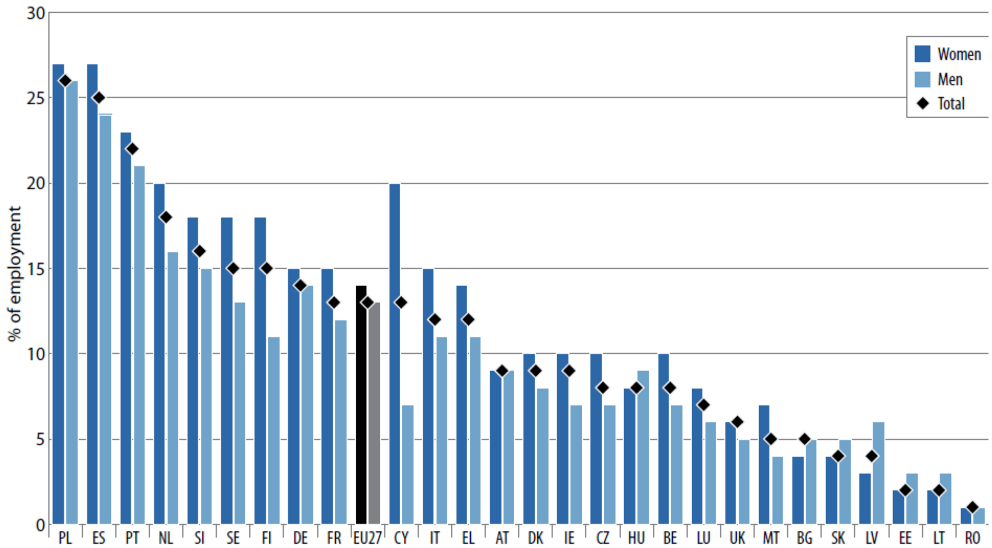
As mentioned before, the growing literature studying the sources of resource misallocation among production units focuses on many reasons to interpret the differences in the size distribution of firms across countries. We next show additional evidence indicating that countries with particularly high fraction of temporary contracts in their labor force have a relatively large share of small firms and lower productivity, suggesting that labor market regulations that stimulate firms to extend the use of temporary may play an important role in shaping the size distribution of firms.

Fixed-term contracts are relatively common in some countries such as Poland and Spain, where 25% or more of employees have such a contract, and Portugal with more than 20% in 2009 (see Figure 3). In contrast, temporary contracts accounts for less than 15% of employees in Germany, and just a bit more than 5% of employees in Luxembourg and UK. Following Boeri (2010b), the 2010 Eurostat Report on Employment in Europe (Figure 4) shows that stricter employment protection legislation (EPL) for permanent contracts tended to be associated with a higher share of temporary employment across EU countries (firms substitute permanent workers

[9] For instance, they find that among European Union countries trade performance differ significantly where Germany is by far the most export oriented, with a share of exports to GDP of 39.9 percent, followed by Italy (23.4 percent), France (21.3 percent), the United Kingdom(17.2 percent) and Spain (16.7 percent). Then they show that German firms tend to be larger and Italian firms smaller than the EU average in all sectors. Furthermore, using firm level microdata collected in 2008 they conduct a counterfactual exercise suggesting that if the industrial structure (in terms of firm size and sectors) of countries such as Italy and Spain were to converge to the structure of Germany, the value of Italian and Spanish total exports would rise considerably, by 37 percent and 24 percent respectively.

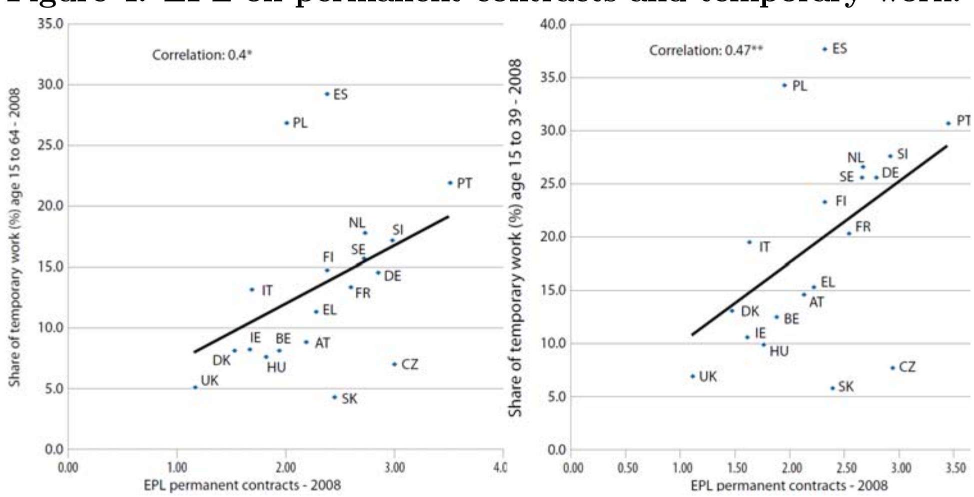
for temporary workers to avoid higher firing costs).

Figure 3: Fixed-term employment for Member States by gender, 2009.



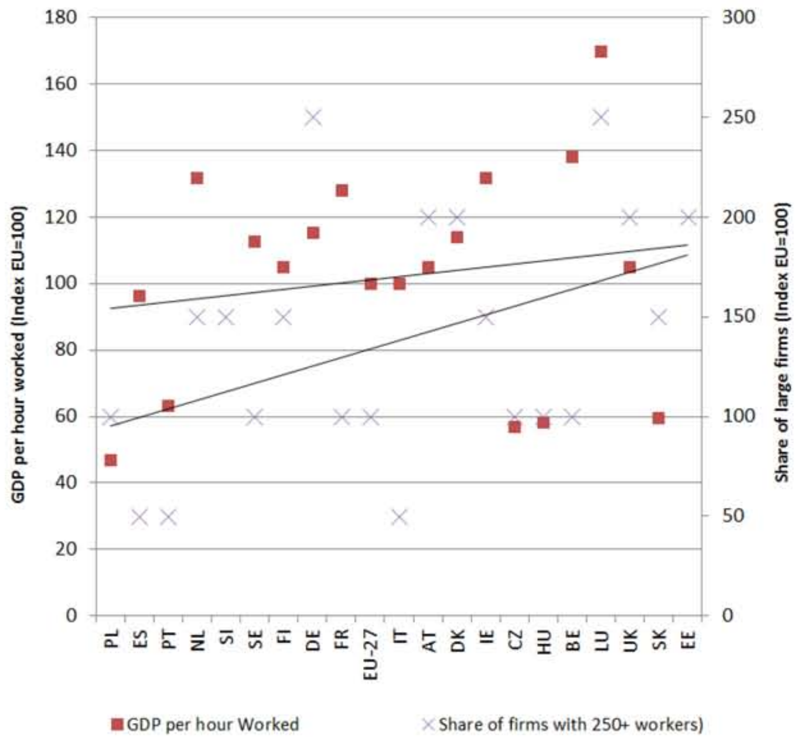
Source: Taken from Eurostat 2009

Figure 4: EPL on permanent contracts and temporary work.



In the horizontal axis of Figure 5 I have ranked countries according to the share of temporary work in decreasing order and the vertical axis contains the share of big firms (indicating an important aspect of the shape of the size distribution of firms) and the GDP per hour worked (an indicator of productivity). It provides preliminary evidence showing that there is a negative relationship between the share of temporary workers and the fraction of big firms as well as with the aggregate productivity across countries. The same results are obtained when using value added per worker and the share of employment in big firms, respectively.

Figure 5: Temporary work and TFP.



Source: Based on data from Eurostat and OECD 2008.

The previous unconditional data analysis is in line with formal empirical evidence on the relationship between DEPL (and temporary work) and the level of TFP documented in other papers, and it represents preliminary evidence about the relationship between temporary work and the size distribution of firms. In addition, it is common knowledge that labor market regulations clearly distinguish the first three countries (in particular in Spain and Portugal) from the rest, and that dual employment protection legislations in those countries have been at the center of the economic debate. Regarding formal empirical evidence, in addition to the literature discussed in the introduction, it is relevant the paper by Dolado, Ortigueira and Stucchi (2012) since it is well known that Spain represents a key case study due to the widespread use of temporary work and the relative strictness of DEPL. Dolado, Ortigueira and Stucchi (2012) argue that since the early nineties Spain has been the EU country with the highest proportion of temporary workers and, in parallel, it has suffered from a drastic productivity slowdown since the mid-1990s. Using a panel of Spanish manufacturing firms they estimate that up to 20% of the slowdown of TFP growth in Spanish manufacturing firms could be explained by the reduction in conversion rates that dual employment protection legislation generates. It is also well known that the lower TPF level that Spain has relative to other European countries is connected with the lower fraction of large firms (or lower share of employees in big firms).[10]

All in all, the unconditional analysis documented in the current paper suggests that countries with labor market regulations that disincentive the relative use of permanent contracts have a higher fraction of temporary work, and a higher fraction of temporary work is associated with a lower fraction of large firms and lower productivity. In addition, there is well documented microevidence for the Spanish economy that points to the dual employment protection legislation as a key determinant for the lower productivity. In the current paper we explore a new specific mechanism linking changes in the strictness of the dual employment protection legislation to aggregate productivity by means of distortions on firm selection and the allocation of resources across firms.

2.2 Model

2.2.1 Key features of the model

In this section we describe the main ingredients of the model, introduce some notation, and explain the induced optimal behavior by agents and the main tradeoffs before going to the model more in detail. We develop a theory of firm dynamics with search frictions and asymmetric firing costs of permanent and temporary workers. Let's now explain each of these components.

First, by firm dynamics we mean the following. There is a continuum of potential entrant firms that upon entry have the same initial productivity and draw a rate of productivity growth, g, from a distribution with cdf $G(g)$. Firms live at most $\hat{J}$ periods, they produce homogeneous goods that are sold in a competitive market and for production they need to employ workers.

Second, regarding the presence of search frictions, just like in Mortensen and Pissarides (1994), we assume that firms and workers have to search for each other and there is a matching technology that relates the probability of workers and employers finding a counterpart in the labor market to the ratio of vacancies to unemployed members of the labor force, denoted by θ. Due to the linear homogeneity of the matching function, $m(\theta)$, job seekers meet firms at the rate $\theta m(\theta)$ which is increasing in θ. Following Felbermayr, Prat and Schmerer (2011), the cost of posting vacancies is proportional to a function c_v, so that recruiting x workers entails spending $c_v x$, where c_v is the cost of a vacancy divided by the probability of filling the vacancy,

[10] See FEDEA (2009), and Pol Antras in NeG (2010), http://www.fedeablogs.net/economia/?p=7234.

$c_v = c/m(\theta)$. Whereas marginal recruitment costs are increasing at the aggregate level because of congestion externalities, they are exogenous from a firm's point of view. In addition, we assume that there are unemployment benefits, denoted by b.

Third, regarding temporary and permanent workers and the asymmetric firing costs we assume that, temporary workers are employees that remain matched with a firm for a short period of time (they have an exogenous separation rate equal to one) and are dismissed at zero firing cost. Permanent workers are employees that remain matched with a firm until it decides to fire them (they have an exogenous separation rate equal to zero), but have high firing costs, τ_f. In addition we allow permanent workers to have exogenous higher productivity than temporary ones (this is not a fundamental assumption; the results remain the same if we assume that temporary and permanent workers have similar exogenous productivity).

With respect to the timing, after entry a firm observes its productivity growth rate, it decides the optimal age to exit, and decides how many vacancies to post. After matches have taken place, it decides how many workers to hire as temporary or permanent workers. Then firms and workers bargain over wages according to their particular labor contracts (permanent or transitory contracts). There is free entry, thus new firms are born (enter) every period.

In this framework firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. Furthermore, firms with the same productivity level may choose different optimal sizes and fraction of permanent and temporary workers. In order to provide a better interpretation on this optimal firms´ behavior, let's focus on an example in discrete time to analyze the optimal decisions of firms with high productivity growth rate and firms with low productivity growth rate. Let's assume that every firm begins its life with the same productivity level, according to which its optimal size is ten workers. In addition, let's assume that permanent workers are equally productive than temporary workers.

First, let's assume that for a firm with the highest productivity growth rate its optimal employment sequence is $\{10, 15, 20, 25, 30, 35\}$ and thus it lives for $\widehat{J} = 6$ periods. Therefore, if the firm only hires permanent workers then the total cost over the life cycle, net of wages, is $35(c_v + \tau_f)$, where $35c_v$ is the total search cost and $35\tau_f$ is the total firing cost incurred when the firm exit the market. Alternatively, in case the firm decides to hire only temporary workers the net total cost is $135c_v$. Notice that the total search costs in the first case is much lower than in the second case and it may compensate the higher firing costs the firm pay when hiring permanent workers. Therefore, firms with positive growth rate, which survive and last longer in the market, have incentives to hire permanent workers.

Focusing just on the first ten workers that the firm hires right after entry, it can be seen that under a permanent labor contract the costs over the life cycle is $10(c_v + \tau_f)$ versus $\widehat{J}10c_v$ if the firm employs ten workers under a temporary labor contract. Therefore, it is convenient for the firm to use permanent workers whenever $\tau_f < (\widehat{J} - 1)c_v$. If we now focus on the second period that the firm is active in the market, and assuming that it has hired ten permanent workers in the previous period, under the same reasoning it hires five additional permanent workers if $\tau_f < (J-2)c_v$. Notice that the left hand side of the previous inequality is constant while the right hand side decreases as firm ages. Therefore, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. Moreover, firms optimally modify the fraction of permanent workers they hire over their life cycle.

Second, let's focus on a firm with negative productivity growth rate. This example shows that firms with bad prospects on the evolution of their productivity have incentives to hire more temporary workers since they keep reducing production and employment over time and exit the industry relatively soon. Let's assume that a firm's optimal employment path is a sequence of workers $\{10, 5, 2\}$ and exit after $J^* = 2$ periods. Following the same reasoning as in the previous example its easy to notice that; first, it is not optimal for this firm to employ only permanent workers (since some of them will be fired next period); and second, it may be optimal for the firm to employ some permanent workers, for instance 3 permanent workers and 7 temporary workers in the first period, 2 temporary workers in the second period, fire one permanent worker in the third period, and then exit. Therefore, even firms with negative productivity growth rate may find it optimal to hire permanent workers to save on search costs and modify the fraction of temporary and permanent workers along their life-cycle.

The model generates a cross sectional distribution of firms by size and employment composition (different share of temporary and permanent workers). Furthermore, this environment gives rise to rich industry dynamics as firms enter, exit, decide the composition of their labor force over their life cycle and bargain wages for each type of labor contract. Despite the difficulty of the issue we develop a simple model that is analytically tractable to show that high relative firing costs of permanent workers distort firm selection and the allocation of resources across firms. But the main mechanism and insights of the model apply to a more general and realistic model. In the conclusion we argue that, under the presence of search frictions, as long as it takes longer to high productivity firms to exit (they have longer expected life span) and thus it is more valuable for them to hire permanent workers, a more realistic Hopenhayn and Rogerson's (1992) style model capture similar behavior and yields the same results as in the model developed in current paper. We will discuss further the evidence supporting this assumption in the conclusion of the paper.

2.2.2 Value functions for firms and workers

In this section we describe the model more in detail. As mentioned before, since there is a continuum of ex-ante homogeneous firms that are ex-post heterogeneous, there is a cross sectional distribution of firms over the states. As there is free entry and exit and no aggregate uncertainty, by a law of large numbers all aggregate quantities and prices are constant over time. Therefore, when describing the value of firms, employed and unemployed workers we focus on the stationary equilibrium and thus value functions are not indexed by time.

Firms

Firms produce goods according to the production function $z(n^T + \gamma n^P)^\alpha$ where z is the initial productivity level that we assume equal to one, n^T and n^P are the number of temporary and permanent workers, respectively. The parameter α is the span of control parameter ($\alpha < 1$), and γ is the relative productivity of permanent workers. As mentioned before, firms sell their output in a competitive market, and the price is normalized to 1.

We normalize the firing cost of a temporary worker to zero while the firing cost of a permanent worker τ_f , measured in units of output, is positive. We also assume that firms incur the cost τ_t (a training cost) when hiring a permanent worker. We assume that firms pay the training cost when they hire permanent workers (this is without loss of generality because it is optimal to train permanent workers upon hiring them) while the firing cost is paid when firms fire permanent workers. In addition, firms have to pay a per period fixed cost of production, $c_f > 0$, that generates endogenous exit in the model.

Value of an incumbent firm

The state variables of a firm is given by the specific growth rate of productivity, g, and firm's age, a. The aggregate state variable is the labor market tightness, θ. Firms choose a production plan that maximizes discounted lifetime profits. A production plan for a firm with growth rate g and age a is described by the mass of temporary workers, $n^T(g, a)$, and for the mass of hired and fired permanent workers , $h(g, a)$ and $f(g, a)$, respectively, and a time of exit , $J(g)$. As described below, wages are determined through bargaining. In general, the wage that results from bargaining depends on firm characteristics (g and a). For the moment, let us postulate that the bargaining outcome can be summarized by the wage functions

$w^T(g,a)$ and $w^P(g,a)$. Therefore, the value function of an incumbent firm is

$$\begin{aligned} V(g,a;\theta) &= \underset{J(g,a),\{n^T(g,a),f(g,a),h(g,a)\}_a^{J(g,a)\leq \hat{J}}}{Max} \int_a^J \Big[e^{ga}(n^T(g,a)+\gamma n^P(g,a))^{\alpha} \\ & \quad - w^T(g,a)n^T(g,a) - w^P(g,a)n^P(g,a) \\ & \quad - \frac{c}{m(\theta)}\left(n^T(g,a)+h(g,a)\right) - h(g,a)\tau_t - f(g,a)\tau_f\Big] da \\ & \quad - n^P(g,J(g,a))\tau_f - c_f(J(g,a)-a) \end{aligned}$$

$$\begin{aligned} & \dot{n}^P = h(g,a) - f(g,a) \\ & J(g,a) \leq \hat{J} \\ & h(g,a), f(g,a), n^T(g,a), n^P(g,a) \geq 0 \\ & n^P(g,a) \text{ given.} \end{aligned}$$

Thus, a firm with state (g,a) decides how many vacancies to post so as to hire the desired amount of workers $\left(n^T(g,a)+h(g,a)\right)$, incurring a proportional cost of posting vacancies $c_v = c/m(\theta)$ per unit of vacancy posted. It has to pay wages, as well as firing costs and the per period fixed production cost. When the firm dies, at age $J(g,a)$, it has to pay the firing costs to all permanent workers being fired. The first constraint is the law of motion for permanent workers, and the other are just feasibility constraints.

As explained before, when deciding on $n^T(g,a)$, $f(g,a)$, and $h(g,a)$ firms try to balance the higher frequency of search cost expenditures associated to temporary workers with higher firing costs associated to permanent workers, and this trade-off determines the optimal composition of firms' labor force over their life cycle. Firms may find it optimal to hire permanent workers and pay the firing cost τ_f for two reasons. First, to economize on matching costs. Second, permanent workers may be more productive than temporary workers ($\gamma \geq 1$,) where the case $\gamma - 1$ represents the human capital increase due to on the job training paid by the firm.

Entry decision

There is free entry of firms who are ex-ante identical in terms of productivity growth rate and start with the same initial unitary productivity level. In order to enter the industry firms must pay a sunk entry cost, $C_e \geq 0$. After paying C_e they get a draw of productivity growth rate g from the distribution $G(g)$. After observing g they pay the fixed production cost, c_f, post vacancies, pay training costs to the new permanent workers, and bargain over wages with matched workers. Therefore, the value of the expected future discounted profits of a new firm is

$$V_e(\theta) = \int_g V(g;\theta) dG(g). \tag{2.1}$$

Since the function $V(g)$ is increasing in g, the optimal exit decision involves a threshold value g^e such that a firm survive if $g \geq g^e$.

Workers

Here we present the value function for employes and unemployed workers. The appendix 4.1 contains details on the derivations of these equations. The value function of a worker employed in a firm in state (g, a) with a labor contract $s = \{T, P\}$, where s indicates temporary or permanent contract, is given by

$$\rho W(g,a,s) = w^s(g,a) + \delta(g,a,s)\left[U - W(g,a,s)\right] + \left[1 - \delta(g,a,s)\right] \dot{W}(g,a,s),$$

where $\delta(g, a, s)$ denotes the probability that the firm decides to terminate the labor contract. As explained before temporary contracts are terminated with probability 1. The value function of an unemployed worker is

$$\rho U = b + \theta m(\theta) \int \left[W(g,a,s) - U\right] d\mu(g,a,s), \tag{2.2}$$

where $\mu(g, a, s)$ denotes the conditional probability of being matched with a firm in state (g, a) offering a labor contract of type s. Of course, this probability is an equilibrium object that will be formally defined later on and represents the distribution of job offers.

Equilibrium wages

The total surplus generated by a match between workers and firms is split between them. The worker's surplus is equal to the difference between the value of being employed $W(g, a, s)$ by a firm with age a, rate of productivity growth g, under contract s, and the value of being unemployed U. Firm's surplus is simply equal to the marginal increase in the firm's value $\partial V(g, a)/\partial n^P$ (notice that it depends on the labor contract, $s = \{T, P\}$) since each employee is treated as the marginal worker. Following Felbermayr et al (2011) we assume that the outcome of bargaining over the division of the total surplus from the match satisfies the following "surplus-splitting" rule for the case of temporary workers

$$(1-\beta)[W(g,a,T)-U] = \beta \partial V(g,a;\theta)/\partial n^T(g,a) = c_v,$$

and for the case of permanent workers

$$\begin{aligned}(1-\beta)[W(g,a,P)-U] &= \beta \frac{\partial[V(g,a;\theta)-(0-\tau_f n^P(g,a))]}{\partial n^P(g,a)} \\ &= \beta(\lambda(g,a)+\tau_f),\end{aligned}$$

where $\beta \in [0,1]$ denotes the bargaining power of the worker, and $\lambda(g,a)$ is a co-state variable representing the shadow value of a permanent worker (which will be defined formally later on). Applying the envelope theorem for each type of labor contract we get the solutions. Given that we set up the problem of the firm in continuous time, temporary workers obtain the value of being unemployed (the unemployment benefit) for every value of the bargaining power parameter, this is $w^T(g,a) = b\ \forall \beta$, as there is no stock value of a temporary worker to the firm. In the case of permanent worker labor contract, as the threat point of the firm is $\tau_f n^P$, the wage w^P will be higher that the unemployment benefit except in the case in which the bargaining power of the firm is one ($\beta = 0$).

Let's solve for the permanent worker's wage. We know from the value function of a permanent worker that

$$W(g,a,P) = \frac{w^P(g,a)+\delta(g,a,P)U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)},$$

by plugging this equation in the bargaining problem we get

$$\begin{aligned}(1-\beta)[\frac{w^P(g,a)+\delta(g,a,P)U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)}-U] &= \beta(\lambda(g,a)+\tau_f) \\ \frac{w^P(g,a)-\rho U+(1-\delta(g,a,P))\dot{W}(g,a,P)}{\rho+\delta(g,a,P)} &= \frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f),\end{aligned}$$

thus we have

$$w^P(g,a) = \frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)(\rho+\delta(g,a,P)) - (1-\delta(g,a,P))\dot{W}(g,a,P) + \rho U.$$

From the value function of an unemployed worker we have

$$\rho U = b+\theta m(\theta)\left[\int \underbrace{(W(g,a,T)-U)}_{=0}d\mu(g,a,T)+\int \underbrace{(W(g,a,P)-U)}_{=\frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)}d\mu(g,a,P)\right].$$

where the term inside the first integral is equal to zero since we have shown that $(1-\beta)[W(g,a,T)-U]=0$ in the bargaining problem of temporary workers, and the term inside the second integral is $\frac{\beta}{1-\beta}(\lambda(g,a)+\tau_f)$ since $(1-\beta)[W(g,a,P)-U]=\beta(\lambda(g,a)+\tau_f)$ in the bargaining problem of permanent workers. Therefore we have that

$$\rho U = b+\theta m(\theta)\frac{\beta}{1-\beta}\int[\lambda(g,a)+\tau_f]d\mu(g,a,P).$$

By plugging this equation in the wage equation we have found previously we get the following expression for the wage of permanent workers

$$\begin{aligned} w^P(g,a,P) &= \frac{\beta}{1-\beta}\left[(\lambda(g,a)+\tau_f)(\rho+\delta(g,a,P))+\theta m(\theta)\int[\lambda(g,a)+\tau_f]d\mu(g,a,P)\right] \\ &\quad -(1-\delta)\dot{W}(g,a,P)+b, \end{aligned}$$

where remember that $\delta(g,a,P)$ and $\dot{W}(g,a,P)$ are equilibrium objects. Given that these variables are not easy to compute since now on we will follow the analysis for the case in which the firm has all the bargaining power, $\beta=0$, therefore, $w^T=w^P=b$.

2.2.3 Stationary Equilibrium

Before defining a stationary equilibrium, it is convenient to introduce some additional notation and define the law of motion describing equilibrium aggregates. In steady state there will be a constant influx of new firms. To characterize the equilibrium distribution of firms we denote by M the mass of new entrants and we define the following indicator function

$$I(g,a)=\begin{cases} 1 & \text{if } a=J(g) \\ 0 & \text{otherwise.} \end{cases}, \tag{2.3}$$

and the mass of firms of age a and productivity growth g, $X(g,a)$, which satisfies

$$
\begin{aligned}
\frac{\partial X(g,a)}{\partial a} &= -I(g,a)X(g,a) \\
X(g,0) &= M\frac{dG(g)}{1-G(g^e)} \text{ if } g \geq g^e
\end{aligned}
$$

The above law of motion states that the mass of age 0 firms with productivity growth g is given by the mass of entrants times the fraction of businesses that draw productivity growth g among those businesses drawing $g \geq g^e$. As firms aged, the mass of businesses with productivity g stays constant until the optimal exit time $J(g)$. At this age, the mass of businesses with productivity g decreases by 100%.

The probability measure of firms with productivity growth rate $\widetilde{g}$ and age $\widetilde{a}$, hiring workers with contract type $s \in T, P$, are defined as

$$
\begin{aligned}
\mu(\widetilde{g},\widetilde{a},P) &= \frac{X(\widetilde{g},\widetilde{a})h(\widetilde{g},\widetilde{a})}{\int_{g\geq g^e}\int_0^{J(g)} X(g,a)h(g,a)dgda}, \\
\mu(\widetilde{g},\widetilde{a},T) &= \frac{X(\widetilde{g},\widetilde{a})n^T(\widetilde{g},\widetilde{a})}{\int_{g\geq g^e}\int_0^{J(g)} X(a,g)n^T(a,g)dadg}.
\end{aligned}
$$

In addition, the probability that a permanent worker in a business with state (g,a) is fired satisfies

$$
\delta(a,g,P) = \begin{cases} 1 & \text{if } a = J(g) \\ \frac{f(g,a)}{n^P(g,a)} & \text{if } a < J(g). \end{cases} \tag{2.4}
$$

A *stationary equilibrium* is a list of firm decisions rules on temporary and permanent employment $n^T(g,a)$, hiring $h(g,a)$ and firing $f(g,a)$, age of exit $J(g)$, entry threshold g^e, value functions for firms, permanent and temporary employed workers and unemployed workers $(V((g,a;\theta)),\ W(g,a,P),\ W(g,a,T),\ U)$, wage functions $w^T(g,a)$ and $w^P(g,a)$, probability measure for new hires $\mu(g,a,s)$, a mass of entrants M, unemployed N^U and employed N^E, and labor market tightness θ such that:

i) Prices $w^T(g,a) = w^T(g,a) = b$ are given by Nash bargaining (under the assumption $\beta = 0$).

ii) Given $w^T(g,a)$ and $w^T(g,a)$, the firms' production plans $J(g)$, $n^T(g,a)$, $f(g,a)$, $h(g,a)$) are optimal.

iii) Free entry condition is satisfied $C_e = V_e(\theta) = \int_g \max\{0, V(g;\theta)dG(g)\}$.

iv) Laws of motions of different cohorts and entrants as described above,

$$\begin{aligned}\frac{\partial X(g,a)}{\partial a} &= -I(g,a)X(g,a),\\ X(g,0) &= M\frac{dG(g)}{1-G(g^e)} \text{ if } g \geq g^e.\end{aligned}$$

v) The mass of unemployed individuals finding jobs should be equal to the mass of vacancies filled by firms

$$N^U\theta m(\theta) = \int_{g\geq g^e}\int_0^{J(g)} X(g,a)\left[h(g,a)+n^T(g,a)\right]dgda. \tag{2.5}$$

vi) The mass of employed individuals satisfies

$$N^E = 1-N^U = \int_{g\geq g^e}\int_0^{J(g)} X(g,a)\left[n^T(g,a)+n^P(g,a)\right]dgda. \tag{2.6}$$

vii) The mass of workers finding jobs should be equal to the mass of workers being fired (so that unemployment is constant) due to exits and downsizing:

$$\begin{aligned}N^U\theta m(\theta) &= \int_{g\geq g^e}\int_0^{J(g)} X(g,a)\left[f(g,a)+n^T(g,a)\right]dgda \\ &+\int_{g\geq g^e} n^p(g,J(g))X(g,J(g))dg.\end{aligned} \tag{2.7}$$

viii) The mass of entrant firms, M, is consistent with the equilibrium labor market tightness, θ, according to $1 = N^E(M,\theta)+N^U(M,\theta)$.

Proposition 1 *Equilibrium Existence and Uniqueness:* There exist a unique pair (θ^*, M^*) that satisfies the equilibrium definition

Proof Since the newborn firm value, $V(g,a;\theta)$, is monotone decreasing and continuous in θ, thus the expected present discounted value at entry, $V_e(\theta)$, is also monotone decreasing and continuous in θ. Given a value for C_e such that $V_e(\theta = 0) > Ce > V_e(\theta = 1)$, by the intermediate value theorem there exist a unique value θ^* such that $\mathrm{V}_e(\theta^*) - Ce = 0$. By linear homogeneity of $\widehat{X}(\cdot)$, $\widehat{N}^E(\cdot)$ and $\widehat{Vac}(\cdot)$ in M (since policy functions h, f, n^T are invariant in M) then employment and vacancies are continuous and increasing in M and unemployment is decreasing in M, there exist a unique value M^* such that $\theta^* = \frac{M^*\widehat{Vac}(g,a;\theta^*)}{1-M^*\widehat{N}^E(g,a;\theta^*)}$[11]. ■

2.2.4 Solving and characterizing the model for $\beta = 0$

As it was shown previously, the wage for temporary workers does not depend on the bargaining power of the firm and is always equal to the unemployment benefit. On the other hand, the wage for permanent workers depends not only on the bargaining power of the firm but also on equilibrium objects such as the fraction of firms searching for permanent workers, and the optimal firing probability a firm decides on their permanent workers. As it can be noticed in the following solutions, dealing with such complicated expressions would make the analysis less clear and increases significantly the difficulty to focus in the main mechanisms and the economic intuition of the model. Therefore, in this section, and for the rest of the paper, we solve and characterize the solutions of the model for the case in which firms have all the bargaining power. We leave the discussion of the general case for the last sections.

The value of a new born firm with productivity growth rate g is given by

$$\begin{aligned} V(g;\theta) \equiv & \underset{J(g),\{n^T(g,a),f(g,a),h(g,a)\}_{a=0}^{J(g,a)\leq \hat{J}}}{Max} \int_0^J \Big[e^{ga}(n^T(\cdot)+\gamma n^P(\cdot))^\alpha - (w^T n^T(\cdot) + w^P n^P(\cdot)) \\ & -\frac{c}{m(\theta)}[n^T(\cdot)+h(\cdot)] - h(\cdot)\tau_t - f(\cdot)\tau_f\Big] da \\ & - n^P(\cdot,J)\tau_f - c_f J \end{aligned}$$

$$\begin{aligned} &\dot{n}^P = h(g,a) - f(g,a) \\ &J(g,a) \leq \hat{J} \\ &h(g,a), f(g,a), n^T(g,a), n^P(g,a) \geq 0 \\ &n^P(g,0) = 0. \end{aligned}$$

Under the assumption that firms have all the bargaining power both wages, for temporary and permanent workers, are equal to the unemployment benefit, $w^T = w^P = b$. Therefore, the Hamiltonian associated to the above optimal control problem

[11] For more details see the solution algorithm in the appendix.

is

$$
\begin{aligned}
H(g, n^P(\cdot), n^T(\cdot), h(\cdot), f(\cdot), \lambda(\cdot)) \;\equiv\;& \left[e^{ga}(n^T(\cdot) + \gamma n^P(\cdot))^\alpha - b\left(n^T(\cdot) + n^P(\cdot)\right)\right. \\
& \left. -c_v\left(n^T(\cdot) + h(\cdot)\right) - h(\cdot)\tau_t - f(\cdot)\tau_f\right] \\
& -\tau_f n^P(\cdot, J, \cdot) - c_f J + \lambda(g, J, n^p)\left[h(\cdot) - f(\cdot)\right]
\end{aligned}
$$

The Pontryagan's Maximum Principle implies that the necessary and sufficient conditions for an optimal solution to the above problem are given by the control equations (to save space, we do not put the states of the control variables, except for λ):

$$
\frac{\partial H}{\partial n^T} = e^{gt}\alpha(n^T + \gamma n^P)^{\alpha-1} - b - c_v \leq 0 \text{ with } = \text{if } n^T > 0, \tag{2.8}
$$

$$
\frac{\partial H}{\partial h} = -c_v - \tau_t + \lambda(g, a) \leq 0 \text{ with } = \text{if } h > 0, \tag{2.9}
$$

$$
\frac{\partial H}{\partial f} = -\tau_f - \lambda(g, a) \leq 0 \text{ with } = \text{if } f > 0, \tag{2.10}
$$

the multiplier equation

$$
\frac{\partial H}{\partial n^P} = e^{gt}\gamma\alpha(n^T + \gamma n^P)^{\alpha-1} - b = -\dot{\lambda}(g, a) \tag{2.11}
$$

and the state equation

$$
\frac{\partial H}{\partial \lambda} = \dot{n}^P \Rightarrow \dot{n} = h - f, \tag{2.12}
$$

and the transversality conditions

$$
\begin{aligned}
0 \;\leq\;& e^{gJ}(n^T(g, J) + \gamma n^P(g, J))^\alpha - b\left[n^P(g, J) + n^T(g, J)\right] && (2.13) \\
& -c_v\left[n^T(g, J) + h(g, J)\right] - \tau_t h(g, J) - \tau_f f(g, J) - c_f \\
\text{with} \;=\;& \text{ if } J < \hat{J}, \\
\lambda(g, J) \;=\;& -\tau_f && (2.14)
\end{aligned}
$$

The optimality conditions can easily be interpreted. Equation (2.8) states that when firms hire temporary workers, they equate the marginal product of temporary workers to the cost of hiring temporary workers (the wage rate plus the vacancy cost). Firms that do not hire temporary workers exhibit a marginal product of temporary workers below their hiring cost, thereby equation (2.8) holding with inequality. The co-state variable λ represents the shadow value of a permanent worker. Firms hiring permanent workers, equate the marginal value of permanent workers to the sum of recruiting and training costs (see (2.9)). The shadow value of a permanent worker decreases over time ($\dot{\lambda} < 0$). Intuitively, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. The transversality condition (2.14) states that at the end of the match, the value of a permanent worker is equal to $-\tau_f$. The firing decision (2.10) ensures that the value of a permanent worker cannot decrease below $-\tau_f$. Equation (2.11) and $\dot{\lambda} < 0$ imply that the marginal product of permanent workers is above the wage (b) paid to permanent workers. Firms exit at the maximum possible age ($\hat{J}$) when profits are positive at the end of the life cycle. Otherwise, firms exit when profits become equal to zero (see equation (13)). As we shall see, firms exit at $J < \hat{J}$ only if $g < 0$.

Characterizing firms' decisions.

In this section we show how the model works by providing a characterization of the optimal composition of the labor force for each firm's type as well as the optimal age at exit. Besides, we show that there is a particular productivity growth rate $g^* < 0$ below which firms only use temporary labor contracts over their life cycle and above which firms also employ permanent workers and exit at age $\widehat{J}$. In addition we analyze the behavior of firms with zero productivity growth rate (that just hire permanent workers). Once we show how the model works, in the following sections we analyze the general equilibrium effects that changes in the labor market regulations have on the size distribution of firms and aggregate productivity.

We have assumed that the production function is of the form

$$f(n^T + \gamma^P) = (n^T + \gamma n^P)^\alpha. \tag{2.15}$$

Let's make the following two assumptions:

$$\textit{Assumption 1.} \qquad (1-\alpha)\left[\frac{\alpha}{b+c_v}\right]^{\frac{\alpha}{1-\alpha}} > c_f$$

$$\textit{Assumption 2.} \qquad \hat{J} > \frac{c_v + \tau_t + \tau_f}{(b+c_v)\gamma - b}$$

Assumption 1 ensures that firms are profitable at age 0 so that the model economy features a non-trivial equilibrium with production. In characterizing the behavior of firms, we find it convenient to partition firms in two groups depending on whether they ever hire permanent workers or not. The first group is comprised by firms whose rate of growth g is higher than the threshold value g^*:

$$g^* \equiv (1-\alpha)\frac{(b+c_v)\gamma - b}{c_v+\tau_t+\tau_f} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0 \tag{2.16}$$

where $g^* < 0$ follows from Assumption 1. On the other hand, firms with low productivity growth ($g < g^*$) do not hire permanent workers.

Using the fact that permanent workers are only hired if their marginal product is above the wage rate (b), (2.11) implies that the shadow value of permanent workers decreases with the age of the firm,

$$\gamma e^{gt}\alpha(n^T+\gamma n^P)^{\alpha-1} - b = -\dot{\lambda} \geq 0 \Rightarrow \dot{\lambda} \leq 0. \tag{2.17}$$

The declining value of permanent workers implies that if a firm does not hire permanent workers at age 0, it will not do it at a later age ($n(g,0)^P = 0 \Rightarrow n(g,a)^P = 0$ for all a). Whether a firm finds it profitable to hire a permanent worker at age 0, depends on its expected lifetime at birth. Intuitively, if the expected lifetime is long enough, the firm can recoup the fixed cost of hiring a permanent worker. The expected life of a firm at birth is (weakly) increasing in its rate of productivity growth (g). Assumption 2 implies that firms with positive productivity growth have incentives to hire permanent workers at age 0 ($n^P(g,0)) > 0$).

Case I: Firms with low productivity growth ($g < g^*$). As it was explained in previous sections, the value of a permanent worker decreases as the horizon of the contractual relationship diminishes. In particular, there are firms with such a low productivity growth rate that find it optimal to exit relatively soon, thus they do not hire permanent workers at all. This is the case we analyze here. We start by solving the optimization problem under the assumption that $n^P(g,0) = 0$. We then find restrictions in the parameter space so that the optimal solution has this property.

The optimal amount of temporary workers follows from (2.8):

$$n^T(g,a) = \left[\frac{\alpha e^{ga}}{b+c_v}\right]^{\frac{1}{1-\alpha}}. \tag{2.18}$$

Operating profits at age a satisfy:

$$\begin{aligned} \pi(g,a) &= e^{gt}f(n^T(g,a)) - (b+c_v)n^T(g,a) - c_f, && (2.19)\\ &= (1-\alpha)e^{\frac{ga}{1-\alpha}}\left[\frac{\alpha}{b+c_v}\right]^{\frac{\alpha}{1-\alpha}} - c_f. && (2.20) \end{aligned}$$

Using (2.20) it follows that firms make positive profits at age 0 ($\pi(g, 0) > 0$) if Assumption 1 holds.

The firm shuts down if $\pi(J) = 0$ for some $J < \hat{J}$. Solving for J we obtain the age of exit as a function of productivity growth (g):

$$J(g) \equiv \frac{(1-\alpha)}{g} ln \left[\frac{c_f}{1-\alpha} \left(\frac{b + c_v}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \right]. \tag{2.21}$$

The value of a permanent worker at age J is equal to $-\tau_f$ (see equation (2.14)). Using equations (2.8) and (2.11), we can obtain an expression for $\lambda(g, a)$:

$$\lambda(g, a) = \lambda(g, J) - \int_a^J \dot{\lambda} dj \tag{2.22}$$

$$= -\tau_f + [(b + c_v)\gamma - b](J - a) \tag{2.23}$$

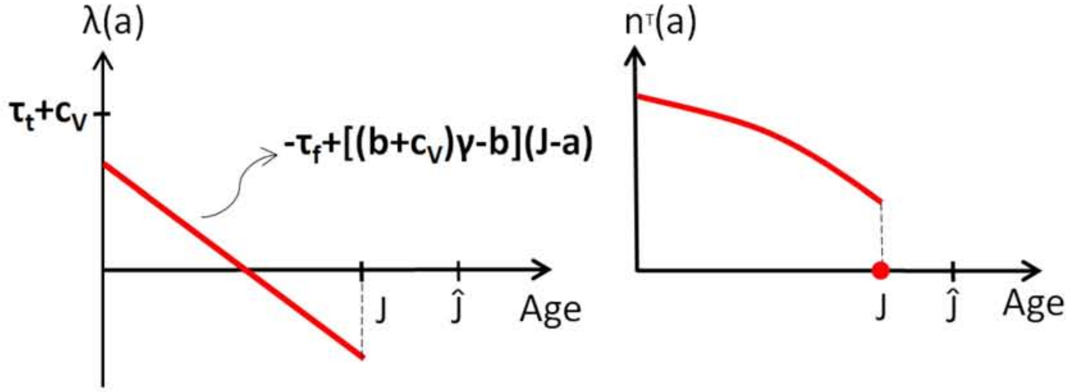
Note that when $\lambda(g, 0) < c_v + \tau_t$ (2.9) implies that $h(g, 0) = 0$ so that the initial assumption of $n^P(g, 0) = 0$ holds true. Using (2.23) it follows that $\lambda(g, 0) < c_v + \tau_t$ holds if

$$J < \frac{c_v + \tau_t + \tau_f}{(b + c_v)\gamma - b} \tag{2.24}$$

Note that the condition in equation (2.24) is fairly easy to interpret when $\gamma = 1$. In this case, firms do not hire permanent workers at age 0 when the the optimal exit age is such that $J < \frac{c_v+\tau_t+\tau_f}{c_v}$, which implies that the cost of hiring one temporary worker for J periods ($c_v J$) is lower than the cost of hiring one permanent worker ($c_v + \tau_t + \tau_f$). Obviously, this condition will be violated when the optimal exit age J is large enough.

Similar interpretation follows when $\gamma > 1$. Firms do not hire permanent workers at age 0 when the cost of hiring one temporary worker for J periods ($J(b + c_v)$) is lower than the cost of hiring one effective permanent worker for J periods ($\frac{c_v+\tau_t+\tau_f+Jb}{\gamma}$). Figure 6 presents the evolution of the shadow value of a permanent worker as well as the dynamics of employment for temporary workers. Since the optimal path value for hiring, firing and employment for permanent workers is zero, we do not plot them.

Figure 6: Dynamics of $\lambda(g,a)$ and $n^T(g,a)$ for $g < g^*$.



Combining (2.21) and (2.24) we find that firms do not hire permanent workers when the rate of productivity growth (g) is such that:

$$\frac{(1-\alpha)}{g} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] < \frac{c_v+\tau_t+\tau_f}{(b+c_v)\gamma-b} \tag{2.25}$$

Rewriting the last expression and using the fact that $g < 0$ we obtain[12]

$$g < g^* \equiv (1-\alpha)\frac{(b+c_v)\gamma-b}{c_v+\tau_t+\tau_f} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] \tag{2.26}$$

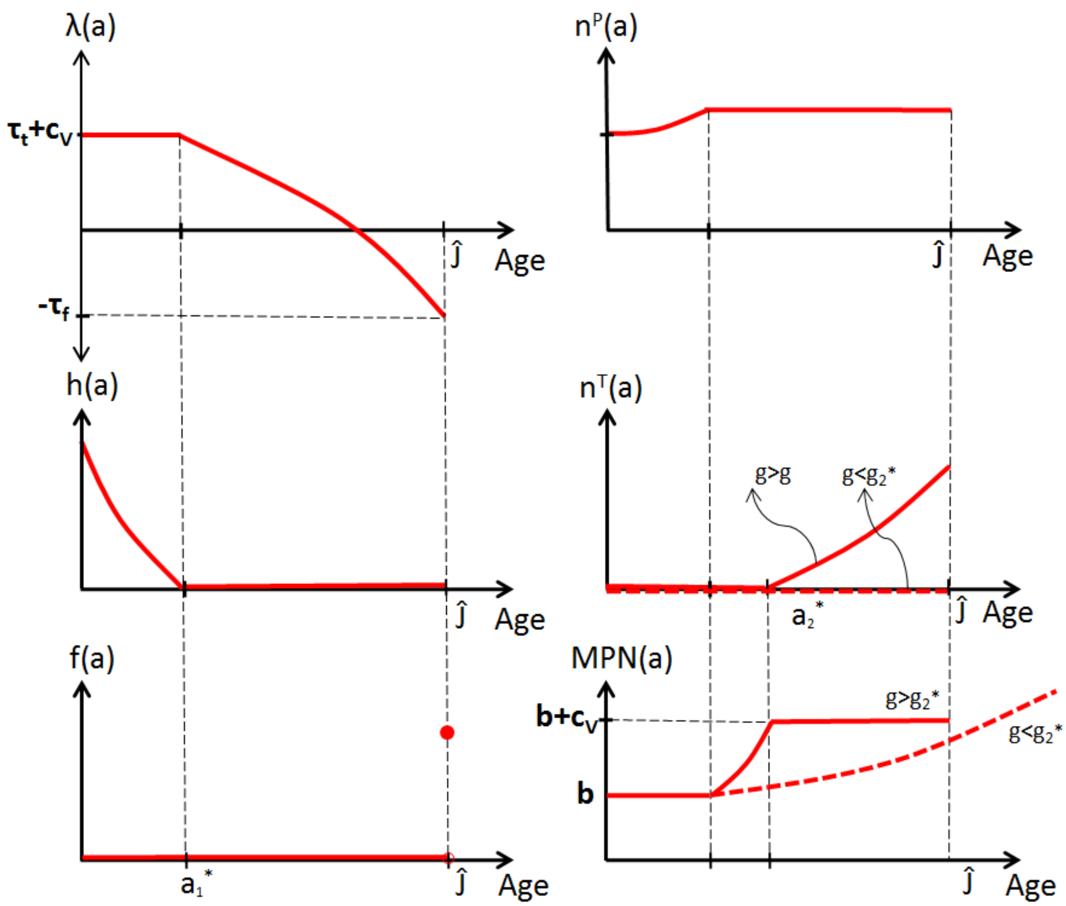
Hence, we have shown that firms with $g < g^*$ do not hire permanent workers. Moreover, Assumption 2 and equation (2.21 imply that firms with productivity g^* exit before the terminal period since $J(g^*) < \hat{J}$ and the optimal age at exit $J(g)$ increases with the rate of productivity growth.

Case II: Firms with high productivity growth ($g > g^*$)**.** In this case we consider firms whose productivity growth is sufficiently high enough, even though it could be negative, to make them profitable to hire permanent workers at age 0 ($n^P(g,0) > 0$). These are firms that last longer in the marker than the ones in Case I. These firms can further be partitioned in two groups: Those who only hire permanent workers at age 0 and those who hire permanent workers for a period of time after birth of the firm. The second group of firms are those with $g > 0$.

[12] Assumptions A2 implies that firms with $g > 0$ will find it optimal to hire permanent workers at age 0.

Case II-A: $g > 0$. Assumption 1 ensures that $\pi(g,0) > 0$ and $g > 0$ implies that $\pi(g,a) > 0$ for all a. As a result, the optimal exit time satisfies $J = \hat{J}$. Note that equation (2.9) implies that firms hire permanent workers at age a if $\lambda(g,a) = c_v + \tau_t$. Now, since $\lambda(g,a)$ is a continuous function and $\lambda(g,J) = -\tau_f$ we know that there exist some $a_1^* < J$ such that firms do not hire permanent workers for all $a \geq a_1^*$. Intuitively, it is not optimal to incur the fixed costs of hiring and training permanent workers when the age of firms is sufficiently close to the age at which they exit the industry. In figure 7 it can be noticed that firms hire permanent workers until age a_1^*, and since then on the employment of permanent workers remains constant until the firm exit the industry. At that point in time the firing policy for permanent workers becomes positive.

Figure 7: Dynamics of main variables for $g > 0$.



Now, notice that (2.9) implies

$$h(g,a) > 0 \Rightarrow \lambda(g,a) = c_v + \tau_t \Rightarrow \dot{\lambda} = 0. \tag{2.27}$$

Substituting $\dot{\lambda} = 0$ in (2.11), we obtain that the marginal product of permanent workers at age a is equal to b. Then, the first order condition on temporary workers (2.8) holds with strict inequality so that $n^T(g,a) = 0$ for all $a \leq a_1^*$. Setting $\dot{\lambda} = 0$ and $n^T(g,a) = 0$ in (2.11), we obtain

$$n^P(g,a) = \begin{cases} \left[\frac{\alpha}{b}\gamma^\alpha e^{ga}\right]^{\frac{1}{1-\alpha}} & \text{for } a \in [0, a_1^*], \\ n^{P*} = \left[\frac{\alpha}{b}\gamma^\alpha e^{ga_1^*}\right]^{\frac{1}{1-\alpha}} & \text{for } a \in [a_1^*, J]. \end{cases} \tag{2.28}$$

For $a \in [a_1^*, J]$ we have $h(g,a) = 0$ so that (2.9) holds with inequality. After age $a > a_1^*$, the marginal product of labor rises at a rate g until it reaches the value of $c_v + b$. Let's denote by a_2^* the age at which the marginal product of labor becomes equal to $b + c_v$. At this age, it becomes profitable to hire temporary workers and the FOC with respect to temporary workers hold with equality. The threshold age a_2^* is obtained from

$$a_2^* \equiv \frac{1}{g} ln \left[\frac{c_v + b}{\alpha}\left(\gamma n^{p*}\right)^{1-\alpha}\right]. \tag{2.29}$$

Defining the threshold growth rate of productivity

$$g_2^* \equiv \frac{1}{\hat{J}} ln \left[\frac{c_v + b}{\alpha}\left(\gamma n^{p*}\right)^{1-\alpha}\right], \tag{2.30}$$

it follows that firms with productivity growth $g < g_2^*$ do not hire temporary workers ($n^T(g,a) = 0$ for all a) since $a_2^* > \hat{J}$ so that they do not reach age a_2^*. Firms with $g > g_2^*$ live beyond age a_2^* and hire temporary workers during the period $a \in [a_2^*, \hat{J}]$. In Figure 8, the last two panels on the right, the optimal path for the hirings of temporary workers, $n^T(a)$, and the marginal product of labor are indicated by the dashed lines (case in which $g < g_2^*$), while the solid lines indicate the case for firms with $g > g_2^*$. For the case of firms with $g > g_2^*$, the optimal amount of temporary workers $n^T(g,a)$ is obtained by solving (2.8) with equality and setting $n^P(g,a) = n^{P*}$.

We close the characterization of the firm problem for firms with $g > 0$, by showing how to obtain the threshold age a_1^* (the values of a_2^* and n^{P*} are expressed in terms of a_1^*). To this end, we use

$$-\tau_f = \lambda(g,J) = \lambda(g,0) + \int_J^0 \dot{\lambda} da \tag{2.31}$$

If the firm does not hire temporary workers during its life cycle ($g < g_2^*$ so that

$a_2^* > \hat{J}$), a_1^* is obtained by solving the following equation

$$\begin{aligned} -\tau_f &= c_v + \tau_t + \int_0^{a_1^*} 0dt + \int_{a_1^*}^{J} (b - e^{gt}\alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1})dt & (2.32) \\ &= c_v + \tau_t + b(J - a_1^*) - \alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1} \frac{1}{g} \left(e^{gJ} - e^{ga_1^*}\right), & (2.33) \end{aligned}$$

where we have used (2.11), (2.31), and the fact that $\lambda(g,a)$ is constant while $h(g,a) > 0$.

If the firm hires temporary workers at the end of its life cycle ($g > g_2^*$ so that $a_2^* < \hat{J}$), a_1^* is obtained by solving the following equation

$$\begin{aligned} -\tau_f &= c_v + \tau_t + \int_0^{a_1^*} 0dt + \int_{a_1^*}^{a_2^*} (b - e^{gt}\alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1})dt & (2.34) \\ &\quad + \int_{a_2^*}^{\hat{J}} (b - e^{gt}\alpha\gamma \left(\gamma n^{P*} + n^T(g,t)\right)^{\alpha-1})dt & (2.35) \\ &= c_v + \tau_t + b(a_2^* - a_1^*) - \alpha\gamma \left(\gamma n^{P*}\right)^{\alpha-1} \frac{1}{g} \left(e^{ga_2^*} - e^{ga_1^*}\right) & (2.36) \\ &\quad + b(\hat{J} - a_2^*) - \left[\gamma(b + c_v) - b\right](\hat{J} - a_2^*). & (2.37) \end{aligned}$$

where we have used (2.8), (2.11), (2.31), and the fact that $\lambda(g,a)$ is constant while $h(g,a) > 0$.

Case II-B: $g = 0$. Firms with $g = 0$ hire permanent workers at age 0 and then do not hire any additional worker (neither temporary nor permanent). Assumption 1 implies that these firms exit at the terminal period ($J = \hat{J}$). Equation (2.9) implies that $\lambda(g,0) = c_v + \tau_t$. For $n^P(g,a) = n^P(g,0)$ for all a, equation (2.11) requires that $\dot{\lambda} = \bar{\lambda}$ for some constant $\bar{\lambda} < 0$. We then have

$$\begin{aligned} \lambda(g,a) &= \lambda(g,0) + \int_0^a \dot{\lambda}(g,t)dt = \lambda(0) + \int_0^a \bar{\lambda}dt & (2.38) \\ &= c_v + \tau_t + a\bar{\lambda} & (2.39) \end{aligned}$$

Using the transversality condition $\lambda(g,\hat{J}) = -\tau_f$, we obtain $\bar{\lambda} = -\frac{c_v + \tau_t + \tau_f}{J}$. Equation (2.11) at time 0 implies that the marginal product of labor with respect to permanent workers is such that

$$\gamma^\alpha \alpha n^P(g,0)^{\alpha-1} = b + \frac{c_v + \tau_t + \tau_f}{\hat{J}}. \quad (2.40)$$

Since firms do not hire temporary workers, the FOC with respect to temporary workers at time 0 (equation 2.8) implies that

$$\alpha n^P(g,0)^{\alpha-1} < b + c_v. \tag{2.41}$$

Combining (2.40) and (2.41), it is optimal for firms with $g = 0$ to hire permanent workers at time 0 but not temporary workers when the following parameter restriction applies

$$\frac{1}{\gamma^\alpha}\left(b + \frac{c_v + \tau_t + \tau_f}{\hat{J}}\right) < b + c_v,$$

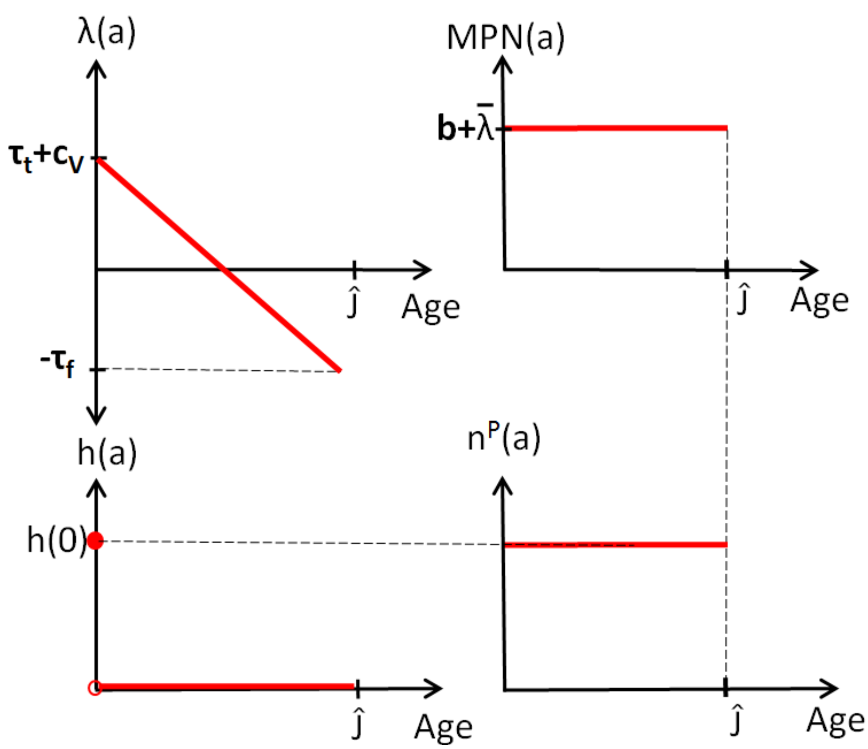
which holds true under Assumption 2.

Using (2.40) to solve for $n^P(g,0)$ we obtain

$$n^P(g,0) = \left(\frac{\gamma^\alpha \alpha J}{b\hat{J} + c_v + \tau_t + \tau_f}\right)^{\frac{1}{1-\alpha}}. \tag{2.42}$$

Figure 8 presents the optimal path for the shadow value of hiring permanent workers, the hirings and total employment of permanent workers, and its marginal productivity.

Figure 8: Dynamics of main variables for $g = 0$.



Case II-C: $g^* < g < 0$. Firms with productivity growth rate $g^* < g < 0$ hire permanent workers in period 0 and exit at age $J < \hat{J}$. From equation (2.9) we have that the shadow value of permanent workers is such that $\lambda_0 = c_v + \tau_t$. Permanent employment remains constant up to a period time in which the growth rate declines enough so as to induce the firm to begin firing permanent workers. Formally, $n^P(g,a) = n^P(g,0)$ for $t \in [0, a_f^*]$, for some $a_f^* > 0$. The shadow value of permanent workers decrease and may reach the value of $-\tau_f$ at age $a_f < J$ (see Figure 9). To put it differently, firms start firing workers before the age of exit and there is a period at the end of the life cycle in which the FOC (2.10) holds with equality ($f(g,a) > 0$ for $a \in [a_f, J]$). Formally, define $a_f \equiv min_a \lambda(a) = -\tau_f$. Note that a_f is the first age at which $\lambda(g,a) = -\tau_f$.

Firms start firing workers before exiting. For $a \geq a_f$,

$$f(g,a) > 0 \Rightarrow \lambda(g,a) = -\tau_f \Rightarrow \dot{\lambda} = 0. \tag{2.43}$$

Using (2.11) we have that permanent employment, before firings take place, is given by

$$n^P(g,a) = \left[\frac{\alpha\gamma^\alpha e^{ga}}{b}\right]^{\frac{1}{1-\alpha}} \text{ if } a \geq a_f \tag{2.44}$$

The optimal age to exit is obtained by solving for J the following equation

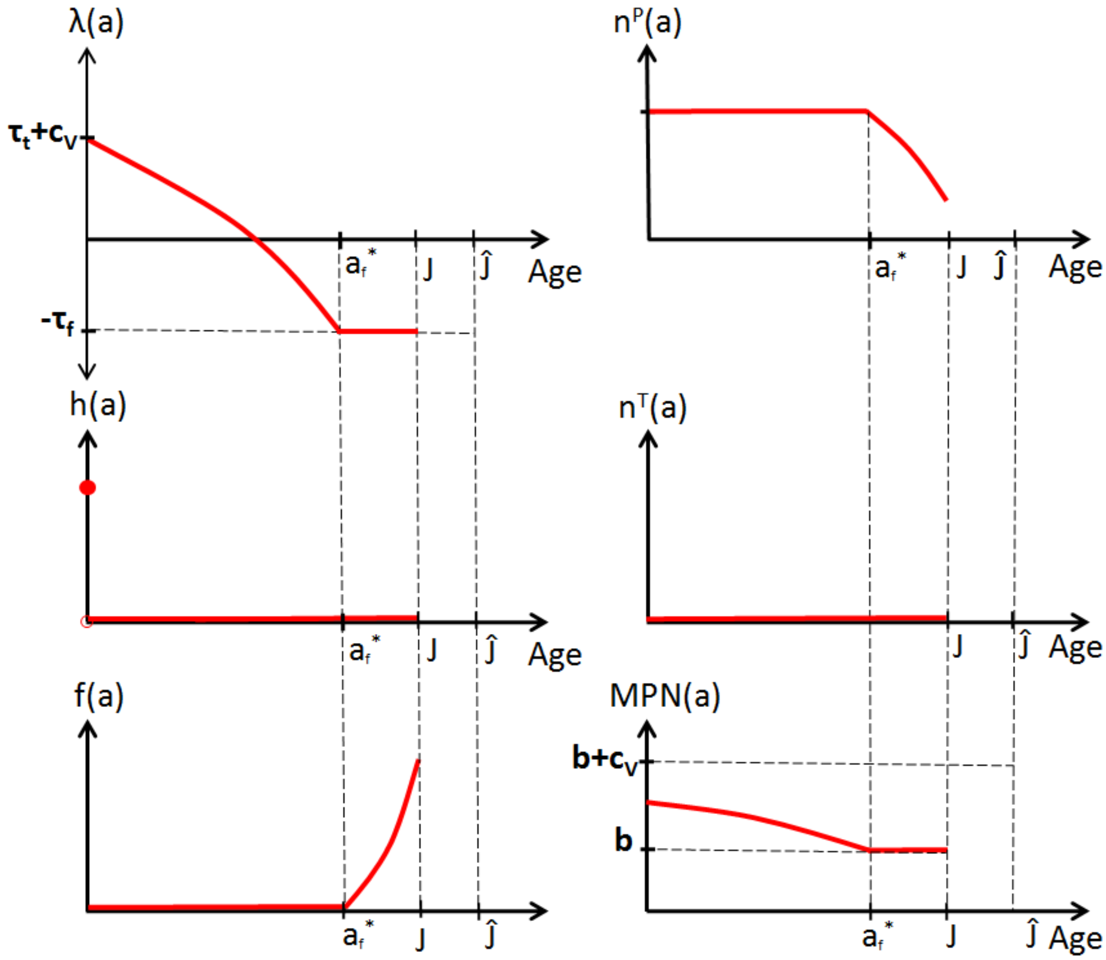
$$\pi(g,J) = e^{gJ}\left[\gamma n^P(g,J)\right]^\alpha - bn^P(g,J) - c_f = 0, \tag{2.45}$$

where $n^P(g,J)$ is obtained from (2.44). The value of a_f is obtained from

$$\begin{aligned}
-\tau_f &= \lambda(g,a_f) = \lambda(g,0) + \int_0^{a_f} \dot{\lambda} dt && (2.46)\\
&= c_v + \tau_t + \int_0^{a_f}\left(b - e^{gt}\alpha\gamma\left[\gamma n^P(g,a_f)\right]^{\alpha-1}\right)dt && (2.47)\\
&= c_v + \tau_t + ba_f - \alpha\gamma\left[\gamma n^P(g,a_f)\right]^{\alpha-1}\frac{e^{ga_f}-1}{g}, && (2.48)
\end{aligned}$$

where $n^P(g,a_f)$ is obtained from (2.44). Figure 9 shows the dynamics of all relevant variables. The shadow value of permanent workers is positive when the firm is born and then declines over time, as the marginal product of labor decreases, while the employment of permanent workers remains constant up to age a_f. At that point in time the firm starts firing permanent workers and, the marginal product of labor remains constant and the firm reduces its size up to the optimal age to exit, J.

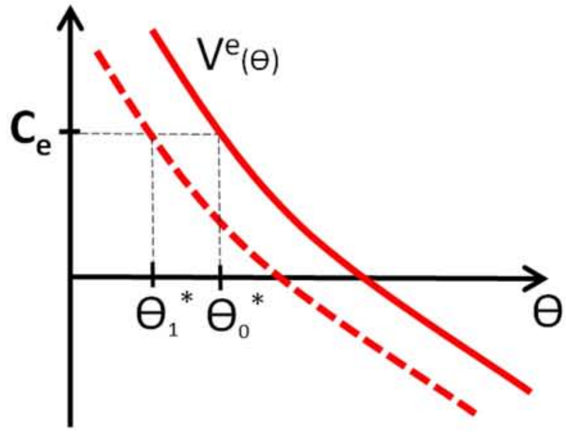
Figure 9: Dynamics of main variables for $g^* < g < 0$.



2.2.5 Analyzing the impact of labor market institutions

In this section we analyze and characterize the effect that an increase in the firing costs of permanent workers have on firm's selection and the employment of temporary and permanent workers of firms with different productivity levels affecting the equilibrium size distribution of firms and aggregate productivity. Formally, following the solution algorithm steps explained in the appendix 4.2, an increase in the firing cost of permanent workers, τ_f, generates a decline in the equilibrium labor market tightness (see Figure 10). This is, as τ_f increases, the expected future discounted profits of a new firm decreases (at the initial labor market tightness, θ_0^*). In Figure 10 this is reflected by a downward shift in the curve representing the expected value at entry. To match the entry cost, C_e, discounted profits need to be higher. Since the expected value at entry, $V_e(\theta)$, is strictly decreasing in θ, this implies that the equilibrium labor market tightness decreases.[13] We denote the new value for the labor market tightness as θ_1^*.

Figure 10: Effect on the equilibrium θ^*.



The intuition behind the previous result is the following. An increase in the firing costs to permanent workers reduces profits (in particular, penalizing more to firms with high productivity growth rate which concentrates a big fraction of employment), inducing to less vacancy posting. Therefore, the labor market tightness declines up to a point in which the value at entry matches again the cost of entry ($V_e(\theta_1^*) = C_e$). As a result, in the new situation there are less vacancies relative to the unemployed workers in the economy.

Furthermore, recall that firms with $g < g^*$ do not hire permanent workers, where

$$g^* \equiv (1-\alpha)\frac{(b+c_v)\gamma - b}{c_v + \tau_t + \tau_f} ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0. \tag{2.49}$$

As mentioned before, note that Assumption 1 implies that $g^* < 0$ since $ln\left[\frac{c_f}{1-\alpha}\left(\frac{b+c_v}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0$. It then follows that an increase in τ_f has two effects on g^* that go in the same direction (which are analyzed in detail later on):

a) Partial equilibrium effect: $\frac{\partial g^*}{\partial \tau_f} > 0$. The threshold value of productivity growth at which firms start hiring permanent workers increase. Hence, an increase in firing costs reduces the mass of businesses hiring permanent workers.

b) General equilibrium effect I: An increase in τ_f reduces profits, inducing to less vacancy posting, and a rise in the unemployment to vacancy ratio. In turn, this increases the probability that a firm matches with a worker (reduces c_v

[13] See the solution method in the appendix for further details.

for a fixed vacancy cost c, this is $c_v = c/m(\theta)$ decreases). Thus, we have that $\frac{\partial c_v}{\partial \tau_f} < 0$ and $\frac{\partial g^*}{\partial c_v} < 0$ so that the general equilibrium effect of an increase in τ_f is to increase g^*.

These two forces give incentives to high productivity firms to reduce their size. Thus, both effects act on the intensive margin. In addition to the previous effects there is another general equilibrium force that induces more distortions in the economy. This additional mechanism has an impact on the extensive margin:

c) General equilibrium effect II: Exit. In general equilibrium, an increase in τ_f subsidizes firms with low growth by reducing the costs of filling up temporary jobs. This subsidy distorts the exit decision of firms by encouraging the hiring of temporary workers. The age of shutdown of low growth firms increases (exit margin). On the other hand, the age of shutdown of high growth firms decreases (exit margin).[14]

In what follows, we show that as τ_f increases firms with low productivity growth rates $g < g^*$ expand, but still employ only temporary workers, and live longer. Firms with intermediate productivity growth rates, $g^* < g < 0$, contract and exit earlier. Firms with zero productivity growth contract, and finally the most productive firms, with $g > 0$, contract. As a result, higher firing costs for permanent workers penalizes relatively more to firms with high productivity growth and subsidizes firms with low productivity growth, shifting employment from the first ones, which contract and last shorter in the market, to the second ones, which expand and last longer in the market. A higher relative firing costs for permanent workers play similar role as a size-dependent-policy by distorting firm selection as well as the allocation of resources across firms implying changes in the size distribution of firms and resulting in a lower aggregate productivity.

We now analyze formally the effect of an increase in firing costs to permanent workers in detail. We organize the analysis by focusing on the effects on firms with different productivity growth rates. The derivations of all these cases follow the characterization of firms' optimal decisions made in the previous sections.

Case I. We first focus on firms with low productivity growth. When there is an increase in τ_f it can be seen from equation (23) that the slope of $\lambda(g, a)$ increases while change in the intersection with the vertical axis is ambiguous. Figure 11 documents the dynamics of the main relevant variables for the case of firms with

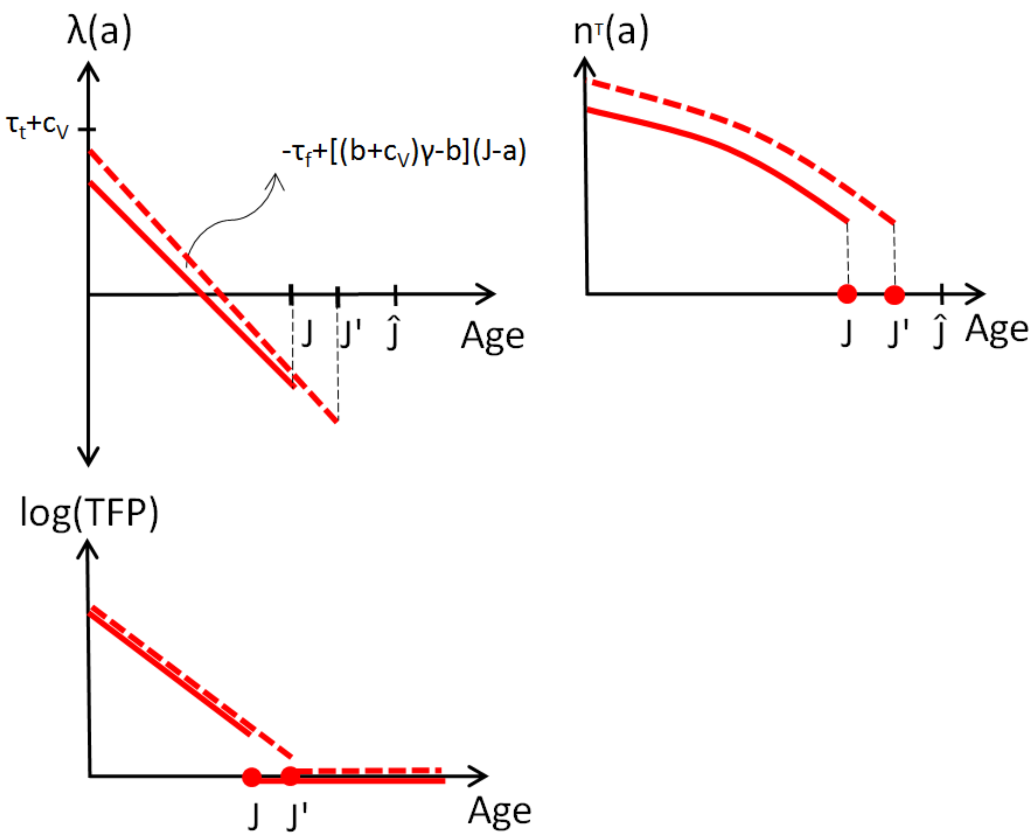
[14] Our model economy assumes that all businesses start with the same productivity and Assumption 1 ensures that businesses are profitable at age 0. Now, if we assume that businesses also differ in terms of their initial productivity then some businesses might not be profitable at age 0. In this case, the hiring subsidy of temporary workers (induced by τ_f) will also distort the entry margin by encouraging entry of businesses with low initial productivity. This extension is left for future research.

$g < g^*$. From equation (21) we have that inefficient firms lasts longer in the market,

$$\frac{\partial J}{\partial \tau_f} = \frac{\alpha \frac{\partial c_v}{\partial \tau_f}}{(b + c_v)g} > 0.$$

In addition, from equation (18) it can be noticed that firms with productivity growth rates $g < g^*$ employ more temporary workers than before (see Figure 11). As these firms last longer in the market their total factor productivity declines further than before the increase in τ_f.

Figure 11: Dynamics of main variables for $g < g^*$.

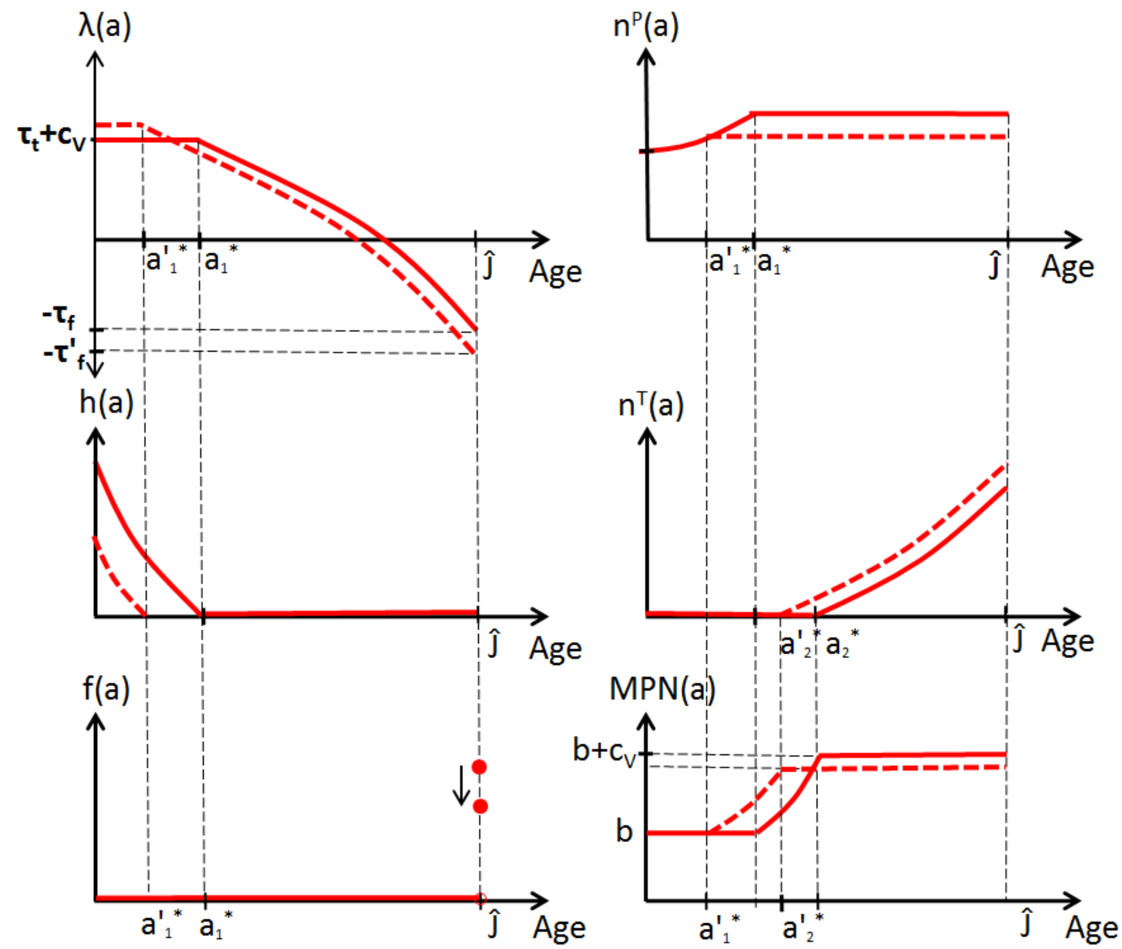


Case II-A. Figure 12 presents the comparative statics results for the case of firms with positive productivity growth rates. Since these firms make positive profits at the beginning of their life-cycle their duration in the market does not change ($\frac{\partial J}{\partial \tau_f} = 0$). An increase in τ_f generates a decline in the LHS of equation (33) and (34), which determines the threshold a_1^*. Therefore, the RHS has to decrease to restore the equality, and since $\frac{\partial n^P(g,a)}{\partial a_1^*} > 0$, thus a_1^* decreases (in Figure 12). The new age threshold is denoted by $a_1^{*\prime}$, and the new firing cost is denoted by τ_f' . Moreover, equations (28), (29), and (30) indicate that firms reduce the employment of permanent workers, $n^P(g,a)$ and $n^P(g,a_1^*)$ (which is reflected by a downward shift in the $n^P(g,a)-$curve in Figure 12), and they stop recruiting permanent workers sooner than before and start hiring temporary workers faster, as also a_2^* and g_2^* decline (notice that $\frac{\partial a_2^*}{\partial a_1^*} > 0$). In addition, as the following expression indicates,

$$n^T(g,a) = \left[\frac{\alpha e^{ga}}{b+c_v}\right]^{\frac{1}{1-\alpha}} - n^P(g,a_1^*),$$

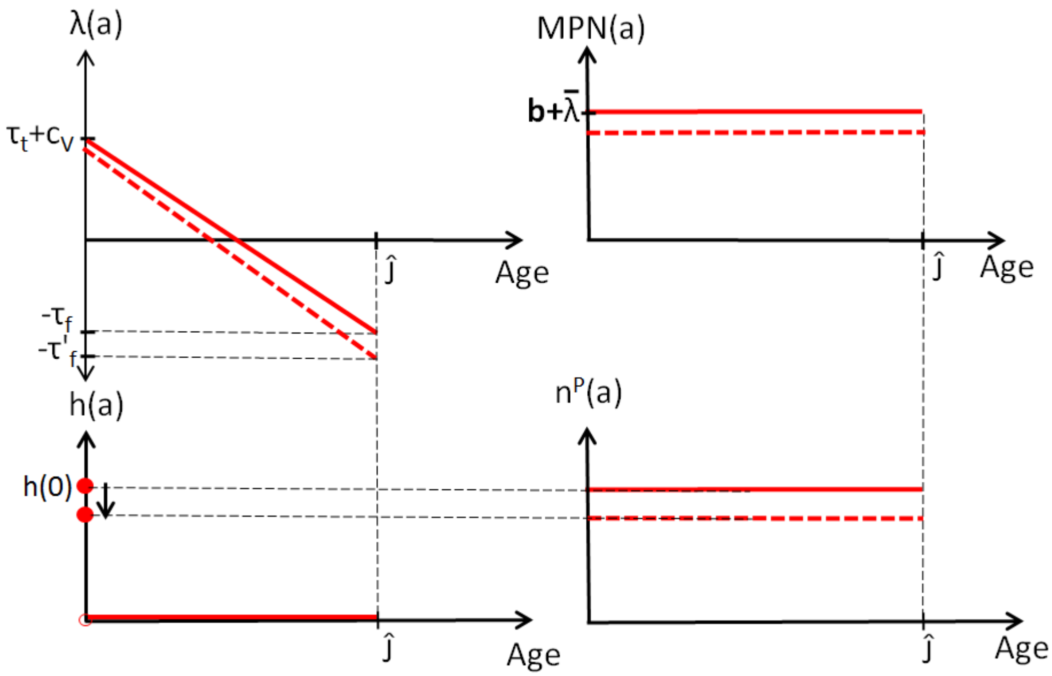
firms increase the employment of temporary workers. This is reflected by an upward shift in the $n^T(g,a)-$curve in Figure 12. As a result, the dynamic path for the marginal productivity of labor is lower than before the increase in the firing cost to permanent workers.

Figure 12: Dynamics of main variables for $g > 0$.



Case II-B. Firms with constant productivity stay active in the market up to period $\widehat{J}$. Since they optimally employ permanent workers to save on search costs they are penalized by the increase in the firing costs τ_f. Thus, they reduce the employment level of permanent workers. Formally, when τ_f increases, the lower labor market tightness induces lower search costs. Thus, equation (39) shows that the shadow value of permanent workers decreases for every point in time. From expression (42) it is clear that the level of permanent employment decreases (see Figure 13).

Figure 13: Dynamics of main variables for $g = 0$.

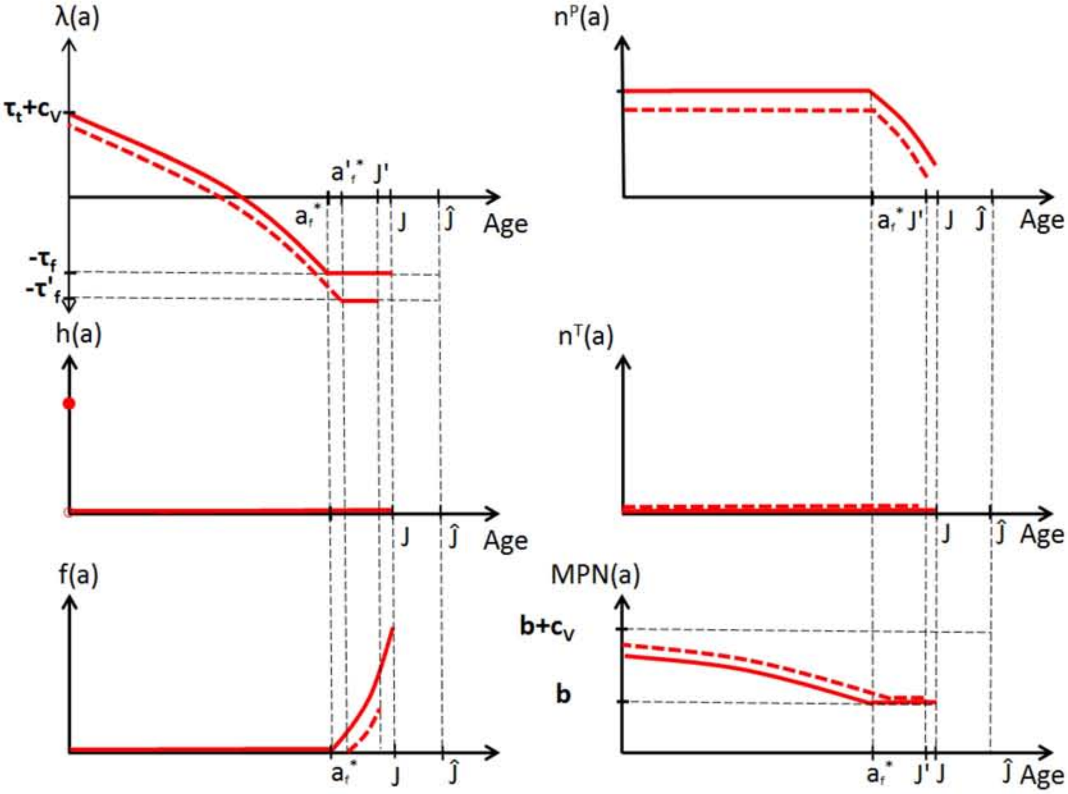


Case II-C. Firms with productivity growth $g^* < g < 0$ also employ permanent workers and thus are also directly affected by an increase in the firing costs to permanent workers. They hire fewer workers than before and exit the industry sooner than before the increase in τ_f. Formally, from equation (48), to match a higher τ_f the optimal age at which the firm starts to fire permanent workers (a_f^*) increases. Notice that, $n^P(g,a) = n^P(g,0)$ for $t \in [0, a_f^*]$. Since the permanent employment, before firings take place, is given by

$$n^P(g,a) = \left[\frac{\alpha\gamma^\alpha e^{ga}}{b}\right]^{\frac{1}{1-\alpha}} \text{ if } a \geq a_f,$$

the number of permanent workers a firm hires is a negative function of the age a_f^*. Therefore the number of permanent workers decreases. Since the shadow value of permanent workers is such that $\lambda_0 = c_v + \tau_t$, and then decreases over time, we have that the new path for $\lambda(g,a)$ is below the previous one. As profits are lower than before for these firms, the optimal age to exit decreases. Figure 14 shows the dynamics of all relevant variables for this case.

Figure 14: Dynamics of main variables for $g^* < g < 0$.



Altogether, as mentioned before, an increase in τ_f leads to a reduction of the number of firms hiring permanent workers (extensive margin) and an increase in the number of firms hiring temporary workers. Moreover, among firms hiring permanent workers, an increase in τ_f reduces the number of permanent workers hired (intensive margin). The general equilibrium effect of an increase in firing costs τ_f implies that firms with $g < g^*$ hire more temporary workers due to the higher probability of filling a vacancy. Thus, an increase in τ_f shifts employment from permanent contracts to temporary contracts and from firms with high productivity growth rate to firms with low productivity growth rate. Total factor productivity decreases because the share of employment in highly productive firms decreases and firms spend less resources in training workers.

2.3 Conclusion and final remarks

Motivated by the fact that countries with strict employment protection legislation of permanent contracts have relatively smaller firms (that concentrate a higher fraction of total employment) and lower aggregate productivity, the current paper develops a model of firm dynamics with search frictions and asymmetric firing costs for temporary and permanent workers. We showed in a very stylized model that firing costs of permanent workers act as size-dependent-policies. Stricter DEPL distorts firm selection as well as the allocation of resources across firms.

As long as it takes longer to high productivity firms to exit (they have longer expected life span) and there are search frictions, the main mechanisms, insights and results of our paper will hold in a more realistic and richer Hopenhayn and Rogerson's (1992) style model. Moreover, there is empirical evidence suggesting that smaller firms have a lower probability of survival and younger firms have a higher probability of exiting.[15] Table 4 shows the pattern of exit rate. Conditional on size younger firms have lower survival rate, and conditional on age, bigger firms have higher survival rate.

Table 4: Firm exit rates by age and size.

	Firm Size (employees)											
Firm Age	**1 to 4**	**5 to 9**	**10 to 19**	**20 to 49**	**50 to 99**	**100 to 249**	**250 to 499**	**500 to 999**	**1000 to 2499**	**2500 to 4999**	**5000 to 9999**	**10000+**
1	37.4	9.3	7.4	6.6	7.0	9.2	8.8	14.9	5.9	0.0	0.0	66.7
2	28.6	7.4	5.3	4.4	4.2	6.6	7.7	7.9	3.1	13.3	20.0	28.6
3	25.8	6.6	4.5	3.8	4.3	3.9	4.0	4.4	0.7	7.1	0.0	0.0
4	23.8	5.6	4.5	3.2	2.5	2.3	1.4	2.9	1.9	7.8	8.7	3.9
5	22.4	5.2	3.8	3.4	3.0	3.6	4.4	8.9	3.7	1.9	2.6	20.0
6 to 10	19.1	4.1	2.9	2.6	2.1	2.0	3.5	3.3	10.2	7.9	2.9	1.4
11 to 15	16.5	3.2	2.3	2.3	2.6	2.2	3.6	4.3	6.8	3.2	9.6	6.9
16 to 20	14.9	2.8	2.0	1.9	1.7	1.8	2.3	1.8	3.2	2.6	4.6	4.5
21 to 25	13.5	2.6	1.9	1.9	2.2	2.1	3.6	2.8	4.5	4.4	4.2	2.2
26+	12.5	2.2	1.9	1.9	2.1	1.7	2.1	2.6	3.5	2.7	5.4	6.3
ALL	19.5	4.0	2.9	2.5	2.3	2.0	2.7	2.9	3.9	4.2	4.6	5.7

Source: US Census Bureau, Business Dynamics Statistics 2010.

Therefore, the data suggests that the mechanism of our paper is empirically relevant. In a more general model in which firms face mean reverting idiosyncratic productivity shocks, it takes longer for large firms to exit the industry. If persistence is very high, as documented in the data, firms with productivity shocks above the mean expects that high shocks today will be around for a long time, thus they are unlikely to exit the industry (they need to accumulate many negative shocks to abandon the industry). In contrast, for small firms (with low productivity level)

[15] For further details see Sutton (1997), Caves (1998), Geroski (1998), Dunne, Roberts and Samuelson (1988, 1989a,b).

few small negative productivity shocks make them exit relatively soon. In addition, if search frictions are present, it is more valuable for large firms with higher survival rate to hire permanent workers. Therefore, the insights of our paper translate into a more realistic Hopenhayn and Rogerson's (1992) style model with search frictions. As an extension to this paper, we are working in such a model and we plan to calibrate it the US economy and perform quantitative analysis to evaluate how dual employment protection legislation (with coherent parameters for firing costs of permanent workers) account for differences aggregate productivity and size distributions of firms between Spain and US. In the new model we also plan to endogenize wages (Nash bargaining) and analyze the effects of wage bargaining at different levels (firm level and collective bargaining).

2.4 Appendix

2.4.1 Derivation of value functions for employed and unemployed workers

Consider discrete time with a period length $\triangle$. Denote by p the probability of finding a job per unit of time so that in a period of length $\triangle$ the probability of finding a job is $\triangle p$. The discount rate per unit of time is ρ. The value of an unemployed worker is

$$\begin{aligned} U_t &= \triangle b + \triangle p e^{-\rho\triangle} \int W(g, t+\triangle, s) d\mu(g,t,s) \\ &\quad + (1-\triangle p)\, e^{-\rho\triangle} U_{t+\triangle} \\ U_t - e^{-\rho\triangle} U_{t+\triangle} &= \triangle b + \triangle p e^{-\rho\triangle} \int \left[W(g,t+\triangle,s) - U_{t+\triangle}\right] d\mu(g,t,s) \\ U_t - [1-\rho\triangle] U_{t+\triangle} &= \triangle b + \triangle p [1-\rho\triangle] \int \left[W(g,t+\triangle,s) - U_{t+\triangle}\right] d\mu(g,t,s), \end{aligned}$$

where the last row makes a Taylor expansion of the term $e^{-\rho\triangle}$ at $\triangle = 0$. Diving both sides of the equation by $\triangle$ and taking the limit as $\triangle \to 0$ gives

$$\dot{U} + \rho U = b + p \int \left[W(g,t,s) - U\right] d\mu(g,t,s), \tag{2.50}$$

and using stationarity of U we can set $\dot{U} = 0$ to obtain the value function of an unemployed worker

$$\rho U = b + \theta m(\theta) \int \left[W(g,t,s) - U\right] d\mu(g,t,s).$$

Similar algebra can be done to obtain the value function of a permanent worker,

$$\begin{aligned} W(g,a,P) &= \triangle_a w^P(g,a,P) + \left[1 - \triangle_a \delta(g,a,P)\right] e^{-\rho\triangle_a} W(g, t+\triangle_a, P) \\ &\quad + \triangle_a \delta(g,a,P) e^{-\rho\triangle_a} U \end{aligned}$$

Rearranging terms,

$$\begin{aligned} W(g,a,P) - e^{-\rho\triangle_a} W(g, a+\triangle_a, P) &= \triangle_a w^P(g,a) \\ -\triangle_a \delta(g,a,n^P) e^{-\rho\triangle_a} W(g,a+\triangle_a,P) &+ \triangle_a \delta(g,a,P) e^{-\rho\triangle_a} U \end{aligned}$$

$$
\begin{aligned}
W(g,a,n^P) - [1-\rho\triangle]W(g,a+\triangle_a,P) &= \triangle_a w^P(g,a) \\
-\triangle_a \delta(g,a,P)e^{-\rho\triangle_a}W(g,a+\triangle_a,P) &+ \triangle_a \delta(g,a,P)e^{-\rho\triangle_a}U
\end{aligned}
$$

Diving both sides of the equation by $\triangle_a$ and taking the limit as $\triangle_a \to 0$ gives

$$
\frac{dW}{da} + \rho W(g,a,P) = w^P(g,a,P) + \delta(g,a,P)\left[U - W(g,a,P)\right], \tag{2.51}
$$

where $\frac{dW}{da} = \frac{\partial W}{\partial a} + \frac{\partial W}{\partial n^P}\frac{dn^p}{da}$, and $\frac{dn^p}{da} = \dot{n}^p$

2.4.2 Solution method

The algorithm to compute the equilibrium is as follows (see Figure 15):

1) Given an initial guess for labor market tightness, θ_0, obtain policy functions $n^T(g,a)$, $h(g,a)$, $f(g,a)$, and optimal age of exit $J(g)$ and entry threshold g^e.

2) Compute newborn firm's value functions $V(g;\theta)$ and the expected value at entry, $V_e(\theta) = \int_g \max\{0, V(g;\theta)dG(g)\}$, and

   - If $V_e(\theta) < C_e \Rightarrow$guess a lower θ by bisection and repeat from point (1).
   - If $V_e(\theta) < C_e \Rightarrow$guess a higher θ by bisection and repeat from point (1).

   When $V_e(\theta^*) \approx C_e \Rightarrow$stop and go to next point.

3) Set the mass of entrants to one, $M = 1$, and use decision rules to compute the measure of firm of different age and growth rate, $\widehat{X}(g,a)$. Compute aggregate employment of permanent workers and aggregate vacancy postings when $M = 1$, denoted by $(\widehat{Vac})$:

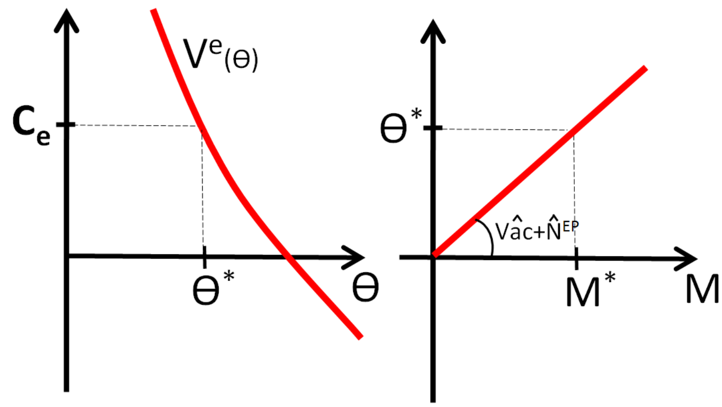
$$
\begin{aligned}
\widehat{N}^{EP}(g,a;\theta^*) &= \int_{g\geq g^e}\int_0^{J(g)} \widehat{X}(g,a)n^P(g,a)dadg \\
\widehat{Vac}(g,a;\theta^*) &= \frac{1}{m(\theta^*)}\int_{g\geq g^e}\int_0^{J(g)} \widehat{X}(g,a)\left[n^T(g,a)+h(g,a)\right]dadg.
\end{aligned}
$$

   Notice that the supra hat in all variables indicates when $M = 1$.

4) Use the linear homogeneity of $\widehat{X}(\cdot)$, $\widehat{N}^{EP}(\cdot)$ and $\widehat{Vac}(\cdot)$ in M to compute the equilibrium measure of entrants, M^*, consistent with the equilibrium labor

market tightness, θ^*, see Figure 15,

Figure 15: Equilibrium θ^* and M^*.



this is, find M^* such that $\theta^* = \frac{M^*\widehat{Vac}(g,a;\theta^*)}{1-M^*\widehat{N}^{EP}(g,a;\theta^*)}$ where $1 - M^*\widehat{N}^{EP}(g, a; \theta^*) = N^U$.

Chapter 3

Size-Dependent Policies and Vertical Integration

3.1 Introduction

In this paper I develop a model in which regulations that restrict the size of establishments (size-dependent policies) distort firms' optimal organization of production and have non trivial-effects on the size distribution of firms and total factor productivity (TFP).[1] The model is motivated by three facts: 1) cross country disparities in income per-capita are mostly accounted for by differences in TFP, 2) there are systematic differences in the organization of production across countries, in particular, developing countries have fewer vertically integrated firms, 3) vertically integrated firms are more productive, bigger and are matched to better suppliers (with high productivity and size).

I develop a dynamic model of an industry with heterogeneous firms interacting as buyers and sellers of inputs, endogenous vertical integration, and market frictions. In the model economy, the vertical industrial structure emerges endogenously as the result of optimal investment decisions that firms undertake. Firms choose whether to integrate to external sellers and use specialized inputs in their production process, or buy homogeneous (standardized) inputs in the market. The use of specialized inputs requires an investment (the acquisition of supplier's plant) and it implies variable cost advantages that depend on both, the productivity of manufacturer and supplier. In addition, vertical integration imposes a higher per period fixed cost of production, which reflects the additional managerial costs of managing two plants. Thus, only high productivity manufacturers (downstream firms) become vertically integrated

[1] Guner, Ventura and Xu (2008) provide real world examples of size-dependent policies including policies that limit the size of manufacturing establishments in India, the size of retail establishments in Japan, and employment protection policies in Italy that only take effect beyond a certain size threshold. In the current paper, size-dependent policies are modelled as taxes on output or labor costs that establishments above a certain size have to pay. For quantitative analysis of small scale reservation laws in India, which is an extreme form of size-dependent policies, see Garcia-Santana and Pijoan-Mas (2012). Garicano, Lelarge, and Reenen (2012) study labor regulations that only bind for firms with more than 50 workers in France.

with high productivity suppliers. In this framework, distortions on production and employment generate a reallocation of resources (employment) from big firms to small firms and act as barriers to vertical integration.

The current paper is related to two literature. First, a large literature has emerged to understand the significant differences in output per capita across countries. Although differences in resource endowments may play a role to explain this fact (e.g. Caselli and Coleman 2006) these differences have been attributed to differences in productivity, and not to factor endowments.[2,3] Such differences in productivity can emerge from distortions that perturb the efficient allocation of resources across production units. A growing body of recent literature has recently focused on distortions that affect the efficient allocation of resources across production units and show how such distortions can have substantial effects on aggregate productivity. At the broadest level, the literature has focused on trade restrictions, financial frictions, and other regulations associated to industrial policy as a source of resource misallocation. For instance, Restuccia and Rogerson (2008) model distortions as firm or plant-specific. Hsieh and Klenow (2009) follow Restuccia and Rogerson and study the impact of misallocation across establishments in explaining productivity in manufacturing in China and India. Furthermore, they, as well as Bartelsman, Haltiwanger and Scarpetta (2013), recover the underlying distortions from observed allocations.[4] Guner, Ventura and Xu (2008), using a Lucas (1978) span-of-control model, consider policies that directly target the size of the establishment such as a tax on factor inputs that establishments with more than given number of employees must pay. When a general configuration of these policies are restricted to achieve a given reduction in average establishment size, they find a substantial reduction in aggregate output per worker. In their set-up, size-dependent policies distort the optimal allocation of resources across production units and move capital and labor from more to less productive establishments. In this paper I focus on how distortions that are firm specific and correlated with the size restrict the growth of efficient firms by affecting the level of vertical integration activity. Size-dependent policies,

[2] Caselli and Coleman (2006) develop a model in which countries with large endowments of skilled labor tend to have lower skill premiums and, consequently, are more likely to pick skill-intensive production technologies.

[3] See King and Levine (1994), Klenow and Rodríguez-Clare (1997), Hall and Jones (1999), and Caselli (2005).

[4] Caselli and Gennaioli (2005), Erosa and Hidalgo (2008), and Buera, Kaboski and Shin (2011), Gilchrist, Sim and Zakrajsek (2013), Caggese and Cunat (2013), Greenwood, Sanchez and Wang (2013), Buera, Moll and Shin (2013), focus on financial frictions as a source of resource misallocation. While Bond, Crucini, Rodrigue and Potter (2013), and Eslava, Haltiwanger, Kugler and Kugler (2013), focus on trade restrictions as a source of resource misallocation. And Restuccia and Rogerson (2008), Guner, Ventura and Xu (2008), Bhattacharya, Guner and Ventura (2013), Gabler and Poschke (2013), Brandt, Tombe and Zhu (2013), Bollard, Klenow and Sharma (2013), Ziebarth (2013), and Oberfield (2013), focus on other regulations associated to industrial policy as a source of resource misallocation. Restuccia and Rogerson (2013) provide a summary of the literature.

by their very nature create a disincentive for firms to be large and limit vertical integration.

Second, there is a big literature that focuses on the organization of production. This literature, which goes back to the seminal paper by Coase (1937), has focused on the scope of the market versus the firm. Since then, important contributions on transaction cost economics and contract theory have been emphasizing the role of transaction costs, asset specificity, supply uncertainty, incomplete contracting, market power and regulation on vertical integration.[5] According to Garfinkel and Hankins (2011), vertical integration is an important proportion of mergers and acquisitions in US (fluctuating from 44.7% to 33.4%). In addition, focusing on vertical integration within industries in the US, Hortaçsu and Syverson (2009, 2013) show that vertically integrated producers are more productive, bigger and are matched to better suppliers (with high productivity and size).[6]

Regarding the organization of production across countries, there is evidence indicating a prevalence of subcontracting arrangements in the developing countries.[7] Maquiavello (2012), discuss the theoretical and empirical literature on vertical integration across countries.[8] Maquiavello (2006) documents a positive correlation between vertical integration across 25 industries in the manufacturing sector and the GDP per capita of each country. He uses the unweighted average of the ratios of value added over output as a proxy of vertical integration. The use of the unweighted average of the ratios of value added over output is a commonly used proxy of vertical integration in the industrial organization literature.[9]. At the firm level, the ratio of value added over output measures the proportion of the production process that is carried out within firm boundaries. A higher value of the index

[5] The literature, at the broadest level, has considered the following perspectives on vertical integration: agency theory articles include Alchian and Demsetz (1972) and Holmstrom (1982); transaction costs theory research includes Williamson (1979); and the references for the property right theory are Grossman and Hart (1986) and Hart and Moore (1990). Gibbons (2005) provides a summary and a comparison of these theories. The most recent surveys include Joskow (2005) and Lafontaine and Slade (2007). Recent theoretical and empirical research on the study of the determinants and effects of vertical integration within and across industries include McLaren (2000), Grossman and Helpman (2002), Antras (2003), Acemoglu et al. (2004) and (2005), Novak and Stern (2007a,b), Ciliberto and Panzar (2009), Legros and Newman (2009) and Gibbons, Holden and Powell (2010).

[6] Hortaçsu and Syverson (2009) show that vertically integrated plants represents 8 to 9 percent of the total plants and account for 25 to 30 percent of employment, in the manufacturing sector in US. Furthermore, when vertically integrating, large and more productive downstream firms choose also large and more productive upstream production units. In addition they find that the fraction of vertically integrated plants increases with size (smallest plants almost never integrate, 7 % of median-sized plants are vertically integrated, and 67 % plants in the top percentile are vertically integrated). Besides, being larger, vertically integrated plants have higher productivity (on average they are 40 % more productive).

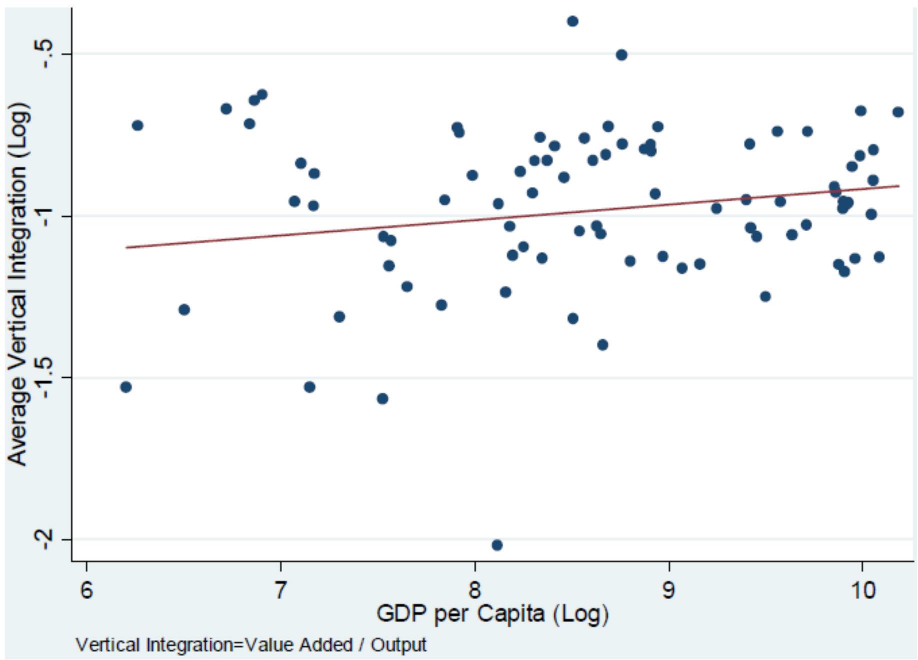
[7] For instance, evidence from the computer industry in Taiwan (Levy 1990), the Guadalajara shoe cluster in Mexico (Woodruff 2002), the Sinos Valley in Brazil (Schmitz 1995), the Tirupur cotton industry in India (Banerjee and Munshi 2004). Andrabi et al. (2006) study the subcontracting arrangements of a large tractor producer in Pakistan.

[8] See also Acemoglu et al. (2009).

[9] See Adelman (1955).

is associated with a higher degree of vertical integration. Figure 1, taken from Maquiavello (2006), shows that there is a higher propensity for firms to vertically integrate in more developed countries, and is consistent with evidence suggesting that subcontracting arrangements are fairly extensive in the developing countries. The ratio of value added over output is 22% higher in developed countries relative to developing coutries (44% versus 33%).

Figure 1: Vertical Integration and GDP Per Capita.



Source: Maquiavello (2006).

Motivated by these facts, I develop a dynamic model of an industry with heterogeneous firms interacting as buyers and sellers of inputs, endogenous vertical integration, and market frictions. The model is calibrated to match selected characteristics of the U.S. manufacturing sector, such us the size distribution of firms and the share of vertically integrated firms. This framework is then used to study the impact of size-dependent policies on the organization of production. I find that a 15% output tax on firms that are above mean level of productivity generates a decline in the fraction of vertically integrated firms from 8.7% to 7%, a decrease in TFP of 2.9%, and an increase of 11% in the mean size of firms. A 15% tax on employment generates a decline in the fraction of vertically integrated firms from 8.6% to 7.1%, a decrease in TFP of 2.4% and a decline of 1.2% in the mean size of firms.

3.2 Model

In this section I develop a dynamic model of an industry with heterogeneous firms interacting as buyers and sellers of inputs and market frictions, in which the vertical industrial structure emerges endogenously as the result of optimal investment decisions that firms undertake. Firms choose whether to integrate to external sellers and use specialized inputs in their production process, or buy homogeneous (standardized) inputs in the market. The model is based on Fossati (2013) that develops a Hopenhayn and Rogerson (1993) style model to study how different vertical relations (outsourcing vs. vertical integration) can emerge within an industry equilibrium model. In the current paper, manufacturers' idiosyncratic productivity is assumed to be constant while in Fossati (2013) manufacturers face mean-reverting idiosyncratic shocks. Furthermore, current model abstracts from outsourcing the production of specialized inputs to external sellers. The first simplification is not a fundamental assumption, in the sense that the main result does not depend on the nature of productivity shocks (whether they evolve over time or not), while the second assumption may just moderate the impact of size-dependent-policies on TFP.

3.2.1 Firms

Technology and static decisions

Manufacturers

Manufacturers produce homogeneous final goods, using n units of labor and m units of intermediate goods (inputs) according to the production function $zy(n, m)$ with decreasing returns to scale, where z is an idiosyncratic productivity shock. This productivity shock is drawn from a density function $g(z)$ upon entry and it is constant (it does not change over the life-cycle of a firm).

There is exogenous exit. Every period firms may receive a negative productivity shock that makes the firm die. This occurs with probability δ. Manufacturers sell their products in a competitive market at price p, which is an equilibrium object (determined by manufacturer's free entry condition) and hire workers in a competitive market at a wage rate w.

Manufacturers must pay a tax on production and employment (distortions are going to be introduced one by one, not simultaneously), (τ_y, τ_n), which are drawn upon entry from $g^{\tau}(\tau_y, \tau_n \mid z)$ and depend on z. Let's assume $\tau = (\tau_y, \tau_n)$. Distortions remain constant over time (more details on distortions will be explained later on). Therefore, manufacturers are characterized by (z, τ_y, τ_n).

When a manufacturer enters the industry, since it is not vertically integrated, it has to obtain its inputs from the market for standardized inputs. In particular, it pays a price p_s to buy standardized inputs. It is assumed that this price is determined by Bertrand competition among suppliers. Once $p_s m$ is paid to a randomly matched supplier, the manufacturer simply produces $zy(n, m)$ units of the final good and pays wages and taxes. Thus it has profit function as

$$\pi(z,\tau) = \max_{n(z,\tau),m(z,\tau)} (1-\tau_y)pzy(n(z,\tau), m(z,\tau)) - n(z,\tau)w(1+\tau_n) - m(z,\tau)p_s,$$

where $n(z,\tau)$ and $m(z,\tau)$ are the quantities of labor and standardized inputs used in production; p, w and p_s are the price of the final good, the wage rate, and the standardized input price, respectively.

After buying m units of input from a supplier, the manufacturer learns the productivity ε of the supplier, where $\varepsilon \sim g^s(\varepsilon)$. One may interpret ε as the quality of the match (a match-specific productivity). Given the (z, ε) pair, the manufacturer has two options for the next period: first, it can simply ignore ε and continue buying standardized inputs from the competitive market for inputs and, conditional on staying in the market, getting matched to a new supplier. It is assumed that productivity of standardized supplier is *iid* over time. Therefore, conditional on staying in the market, this manufacturer will start the period in exactly the same situation (as a not vertically integrated manufacturer), this is buying inputs at a price p_s and learning a new ε.

Second, given (z, ε) the manufacturer can become vertically integrated with the particular supplier and produce in-house specialized inputs. Using specialized inputs entails a variable cost advantage reflected by the function $c(z, \varepsilon)$, where z and ε are complements. In order to become vertically integrated with the supplier it has to pay the supplier a price P^{VI} for its firm, which is the market value of the supplier's plant. By becoming vertically integrated the manufacturer keeps matched with the same supplier's type ε. Thus, once a manufacturer becomes vertically integrated, the state (z, τ, ε) is fixed (remains constant until exit). Vertically integrated producers pay an additional fixed cost of operation C_f^{VI} (the fixed cost of a manufacturer that uses standardized inputs is zero). Thus the vertically integrated manufacturer has profit function as

$$\begin{aligned}\pi^{VI}(z,\tau,\varepsilon) \quad &= \max_{n_{VI}(z,\tau,\varepsilon),m_{VI}(z,\tau,\varepsilon)} (1-\tau_y)pzy(n_{VI}(z,\tau,\varepsilon), m_{VI}(z,\tau,\varepsilon)) \\ &\quad - (n_{VI}(z,\tau,\varepsilon) + m_{VI}(z,\tau,\varepsilon))(w - c(z,\varepsilon))(1+\tau_n) - C_f^{VI}.\end{aligned} \tag{3.1}$$

Suppliers

A supplier has constant returns to scale in labor, and sells $m(z, \tau)$ units of standardized inputs according to its matched manufacturer's derived demand at the competitive price p_s. Suppliers are ex-ante homogeneous but once it matches with a manufacturer the quality ε of the specialized input it is able to produce is realized. In case it does not become vertically integrated with the manufacturer the quality of the match, ε, is *iid* over time. A standardized supplier has profit function as

$$\pi^s(z,\tau) = (p_s - w)m(z,\tau), \tag{3.2}$$

where (z,τ) are its matched manufacturer's states variables. In case the supplier becomes vertically integrated with the manufacturer, it receives $P^{VI}(p, p_s)$ and disappears.

Timing

The timing for a not vertically integrated manufacturer attached to a given tax level τ is as follows. At the beginning of every period, according to its productivity z, it decides how many workers to hire, and how many units of the standardized input to buy. It pays a competitive standardized input price p_s to an ex-ante homogeneous randomly matched supplier for m units of inputs. Given its tax level, after observing ε the standardized manufacturer may decide whether to remain not vertically integrated or to become vertically integrated for the next period. At the end of the period with probability δ the firm exit the industry.

A manufacturer with attached tax level $\tau = (\tau_y, \tau_n)$ that is vertically integrated with a supplier of type ε, and given its productivity z, decides current production, how many workers to hire and the quantity of inputs to produce. At the end of the period with probability δ the firm exit the industry.

Dynamic decisions

Incumbents' value functions

The state variables for an active not vertically integrated manufacturer firm is its productivity z, the quality of its supplier ε, and the tax τ. Thus, assuming stationary (distributions, then prices, do not change over time), a value function for the not vertically integrated manufacturer firm is

$$V(z,\tau,\varepsilon) = \max_{n(z,\tau),m(z,\tau),x'(z,\tau,\varepsilon)} (1-\tau_y)pzy(n(z,\tau),m(z,\tau)) - w(1+\tau_n)n(z,\tau) - p_s m(z,\tau)$$

$$+ \left\{ \underbrace{I_{(x'(\cdot)=NVI)}\beta(1-\delta)\sum_{\varepsilon'} V(z,\tau,\varepsilon')g^s(\varepsilon'),}_{\text{Not vertically integrated}} \right.$$

$$\left. \underbrace{I_{(x'(\cdot)=VI)}\left(-P^{VI}(p,p_s) + \beta(1-\delta)\frac{V^{VI}(z,\tau,\varepsilon)}{1-\beta(1-\delta)}\right)}_{\text{Vertically integrated}} \right\}, \tag{3.3}$$

where $x'(z,\tau,\varepsilon) \in \{NVI, VI\}$ be the decision rules for the vertical state of a not vertically integrated manufacturer that is matched with a particular supplier of type ε.

The value function for a vertically integrated manufacturer is

$$V^{VI}(z,\tau,\varepsilon) = \max_{n_{VI}(z,\tau,\varepsilon),m_{VI}(z,\tau,\varepsilon)} \frac{1}{1-\beta(1-\delta)}[(1-\tau_1)pzy(n_{VI}(z,\tau,\varepsilon),m_{VI}(z,\tau,\varepsilon))$$

$$-(n_{VI}(z,\tau,\varepsilon)+m_{VI}(z,\tau,\varepsilon))(1+\tau_2)(w-c(z,\varepsilon)) - C_f^{VI}]. \tag{3.4}$$

With respect to suppliers, the value function of a not vertically integrated supplier with productivity ε that is matched with a manufacturer with productivity and taxes (z,τ) is

$$W(z,\tau,\varepsilon) = (p_s - w)m(z,\tau)$$

$$+\beta \left\{ \underbrace{I_{(x'(z,\tau,\varepsilon)=NVI)} \sum_{z'}\sum_{\tau'}\sum_{\varepsilon'} W(z',\tau',\varepsilon')J^d(z',\tau')g^s(\varepsilon')}_{\text{manufacturer decides to continue not vertically integrated}} \right. \tag{3.5}$$

$$\left. + \underbrace{I_{(x'(z,\tau,\varepsilon)=VI)}P^{VI}(p,p_s)}_{\text{manufacturer decides to become VI}} \right\}.$$

Entry

We assume that manufacturers cannot enter the industry being vertically integrated. They must pay a sunk entry cost $C_e^d \geq 0$, where d indicates downstream firm. They draw z from $g(z), \tau$ from $g^\tau(\tau \mid z)$, and then match randomly with a supplier according to $g^s(\varepsilon)$. Thus, the value of the expected future discounted profits of a new downstream firm is

$$V_e(p, p_s) = \sum_z \sum_\tau \sum_\varepsilon V(z, \tau, \varepsilon; p, p_s) g(z, \tau) g^s(\varepsilon), \tag{3.6}$$

where $g(z, \tau) = g(z) g^\tau(\tau \mid z)$. Entrants in the input industry have to pay a sunk entry cost $C_e^s \geq 0$, where s indicates supplier or upstream firm, earn p_s and draw ε according to $g^s(\varepsilon)$ and matches randomly with a not vertically integrated manufacturer according to $J(z, \tau)$. Thus, the value at entry for an upstream firm is

$$W_e(p, p_s) = \sum_z \sum_\tau \sum_\varepsilon W(z, \tau, \varepsilon; p, p_s) J(z, \tau) g^s(\varepsilon). \tag{3.7}$$

If a supplier becomes VI then it gets $P^{VI}(p, p_s)$ and disappears.

Idiosyncratic Distortions

As in Bhattacharya, Guner, and Ventura (2011), the idiosyncratic distortions are modeled as output and employment taxes on manufacturers (suppliers do not face taxes since distortions would just affect the equilibrium input price) that are dependent on the initial ability level of the entrant, z. As the size of production that an entrant can operate is (strictly) increasing in his productivity, z, relatively larger firms will be more distorted than smaller ones. Upon entry, each firm with productivity level above a given threshold $\widehat{z}$ pays a tax on production or employment $\tau = \tau_y$ or τ_n. Once an entrant is attached to a particular tax, this tax remains constant over time. We assume that taxes are collected from managers and given as a lump sum transfer to households as an income subsidy.

3.2.2 Households

The problem of the household is identical as the one in Hopenhayn and Rogerson (1993). There is a continuum of identical agents with utility function

$$\sum_{t=1}^{\infty} \beta^t [u(c_t) - \kappa(n_t)],$$

where c_t is the consumption of a final good, $n_t \in \{0, 1\}$ is the labor supply, and $\kappa(\cdot)$ is the disutility of work. This problem can be written as one in which there is a representative agent with utility function

$$\sum_{t=1}^{\infty} \beta^t [u(c_t) - aN_t],$$

where N_t is the fraction of agents employed in period t. As it is well-known (see Rogerson, 1988; and Hansen, 1985), this household problem with indivisible labor, in stationary steady state with constant prices and $1/(1+r) = \beta$, becomes static and can be written in the following way

$$\max_{c,N} \; u(c) - aN_t \quad s.t. \;\; pc \leq wN + \Pi + R,$$

where Π is the aggregate profits, and R is the aggregate tax revenues (a lump sum transfer from firms to the agents). The solution to this problem is $N = L^s(p, \Pi + R)$ that represents the aggregate labor supply.

3.2.3 Stationary Equilibrium

Because there is a continuum of firms that upon entry are subject to idiosyncratic shocks and there is exogenous exit, there is a cross sectional distribution of firms over the states (z, τ, ε) and over different firms' vertical structures. We call Φ the stationary distribution of manufacturers using standardized inputs, and Φ^{VI} and Ξ the stationary distribution of vertically integrated manufacturers and standardized suppliers, respectively. Then, the steady-state stationary equilibrium is standard:

A stationary equilibrium in this model is a list of value functions for manufacturers and suppliers $(V(z, \tau, \varepsilon), V^{VI}(z, \tau, \varepsilon)$, and $W(z, \tau, \varepsilon)$; policy functions $n(z, \tau)$, $m(z, \tau)$, $n_{VI}(z, \tau, \varepsilon)$, $m_{VI}(z, \tau, \varepsilon)$, and $x'(z, \tau, \varepsilon)$; prices p and p_s, w and $P^{VI}(p, p_s)$; invariant measures for manufacturers using standardized inputs Φ, vertically integrated manufacturers Φ^{VI} and suppliers Ξ; an invariant density of not vertically integrated firms looking for a supplier $J(z, \tau)$, a mass of downstream and upstream entrants M^d and M^s, $L^s(p, p_s, w, \Pi + R)$, such that:

i) Given p, p_s, w and $P^{VI}(p, p_s)$, policy functions $n(z, \tau)$, $m(z, \tau)$, $n_{VI}(z, \tau, \varepsilon)$, and $m_{VI}(z, \tau, \varepsilon)$ solve the static production decisions

ii) Given p, p_s, w and $P^{VI}(p, p_s)$, policy function $x'(z, \tau, \varepsilon)$ solve the dynamic decisions of not vertically integrated firms *iii*) Free entry conditions are satisfied for manufacturers

$$V_e(p,p_s) = \sum_z \sum_\tau \sum_\varepsilon V(z,\tau,\varepsilon;p,p_s)g(z,\tau)g^s(\varepsilon), \tag{3.8}$$

and suppliers

$$W_e(p,p_s) = \sum_z \sum_\tau \sum_\varepsilon W(z,\tau,\varepsilon;p,p_s)J(z,\tau)g^s(\varepsilon). \tag{3.9}$$

iv) Market clearing conditions are satisfied in the market for final goods $D^d(p) = S^d(p)$ and in the market for standardized inputs $D^u(p_s) = S^u(p_s)$

vi) Market clearing condition is satisfied for the labor market $L^d(p,p_s,w,M^d,M^s) = L^s(p,p_s,w,\Pi(p,p_s,w,M^d,M^s) + R(p,p_s,w,M^d,M^s))$

vii) Laws of motion of states are consistent with individual decisions (stationary measures Φ,Φ^{VI} and Ξ are fixed points).

3.3 Quantitative Analysis

3.3.1 Calibration

In this section I specify functional forms for firms technology and assign parameter values. Basically, the model is calibrated so that the industry stationary equilibrium matches selected characteristics of the U.S. manufacturing sector taken from the U.S. Census Bureau and Hortaçsu and Syverson (2009). In this calibration I treat the US as an economy with no distortions. Table 1 summarizes the values for the parameters set a priori.

Table 1: Parameters set a priori

Parameters	Definition	Value	
β	Discount factor	0.96	assumed
w	Wage	1	normalized
α	Extent of decreasing returns parameter	0.85	Restuccia Rogerson (2008)
δ	Exit rate	0.10	Bartelsman, et. al. (2003)

The population mass is normalized to one, as well as the wage. I assume an exogenous annual exit rate of 10%, consistent with Bartelsman, Haltiwanger and Scarpetta (2013). In addition, I set a discount factor value $\beta = 0.96$. I assume a Leontieff production function, $y(n,m) = min\{n,m\}^{\alpha}$, where the extent of decreasing returns in the firm level production function is $\alpha = 0.85$, that is the same value used in Restuccia and Rogerson (2008). Direct estimates of establishment level production functions and different calibration procedures points to this value as well.[10,11]

As fixed production costs are zero and firms productivity do not evolve over time, there will be no endogenous entry and exit. The distribution $g(z)$ is chosen so that the invariant distribution of firm size across employment levels matches the data (see Figure 2).[12] Since I have assumed that δ is independent of z, the ratios of establishment types in the total invariant distribution $(\Phi + \Phi^{VI})$ are similar as in the distribution $g(z)$, except for the fact that some firms that find an efficient supplier become vertically integrated and thus expand. Therefore, by calibrating the cost advantage function $c(z,\varepsilon)$ and the fixed cost C_f^{VI} I fit the size distribution of firms and the share of vertically integrated firms.[13]

I assume that the distribution for supplier's productivity $g^s(\varepsilon)$ is the same as the distribution $g(z)$. As in Fossati (2013), with respect to the function $c(z,\varepsilon)$, I assume the following functional form

$$c(z_i,\varepsilon_j) = T\left(\frac{z_i - z_{\min}}{z_{\max} - z_{\min}}\right)^{\gamma}\left(\frac{\varepsilon_j - \varepsilon_{\min}}{\varepsilon_{\max} - \varepsilon_{\min}}\right)^{1-\gamma},$$

which is increasing in z_i and ε_j, with $\gamma \in [0,1]$ and where the subindexes min and max for z and ε indicate the maximum and minimum values for z and ε, respectively. The parameter T is the maximum gain from searching a supplier, for the most efficient manufacturer (being z_n and matched with an ε_n, a supplier reduces the nonsunk cost T) after investing P_{VI} in becoming vertically integrated.

[10] See Pavcnik (2002), Atkenson et al (1996), Veracierto (2001), and Atkenson and Kehoe (2005), among others.

[11] Assuming a Leontieff production function is not a fundamental assumption, since the main results of the paper remain the same. For instance, assuming a Cobb-Douglas production function allows manufacturers to substitute n for m. However, notice that VI would still be discouraged, since a vertically integrated manufacturer cannot avoid paying either taxes by substituting n by m. Looking at the profit function of a vertically integrated manufacturer (Equation 2), after an increase in τ_y it is clear that total production would decline further under such a substitution. After an increase in τ_n, besides declining production further, still the firm pays τ_n for total employment $(n+m)$.

[12] I approximate the distribution $g(z)$ on a grid with 35 points considering a log-spaced grid so as to have more points at lower levels of productivity than at higher levels of productivity.

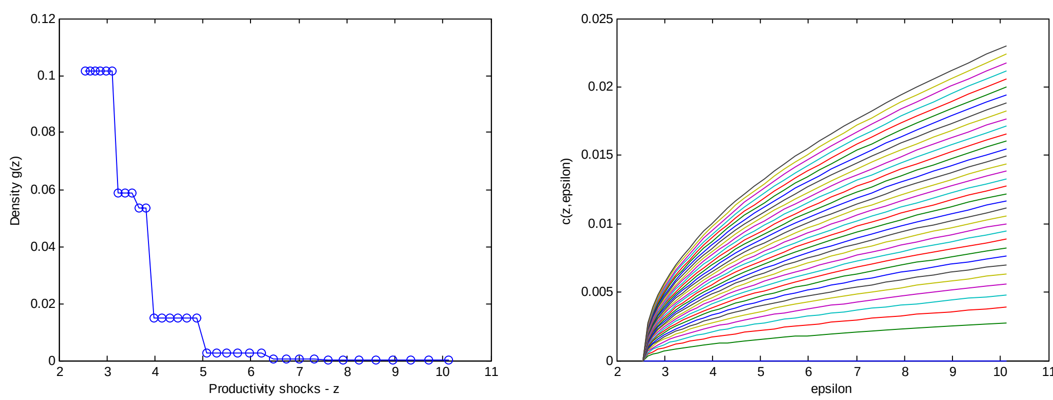
[13] From the data, I only observe the number of firms for a set of employment ranges, therefore, I assume that firms are uniformly distributed in each range so that the cumulative distribution function is a linear interpolation across the points for which we have data.

The parameter γ indicates the relative importance of the manufacturer's type, z, in the decline of variable costs when investments in VI take place. Notice that $c(z, \varepsilon)$ is flexible, in the sense that it allows for the absence of increasing differences. Table 2 presents the values for the calibrated parameters. Figure 2 shows the shape of the function $c(z_i, \varepsilon_j)$ for the parameter values presented above.

Table 2: Calibrated Parameters.

Parameter	Definition	Value
T	Gain from searching for high ε	0.023
γ	Weight of z in c(z,ε)	0.5
C_f^{VI}	Fixed cost of vertical integration	1.6
$g(z)$	Firm size' distribution	see Fig. 2
A	Working disutility	0.68

Figure 2: Density $g(z)$ and cost function $c(z, \epsilon)$



The values for γ, T and C_f^{VI} are chosen so as to fit the size distribution of firms as well as share of vertically integrated firms (8 to 9% of vertically integrated firms). I normalize the final good price to one, $p = 1$, and assume a value for the input price $p_s = 1.16$ consistent with empirical evidence (the model generates a similar average mark-up for final good producers of 17.6%)[14] Given these prices, the level for the sunk entry costs C_e^d and C_e^s were endogenously pinned down by the entry conditions. Finally, the parameter A is chosen to produce an employment to population ratio equal to 0.6 as in Hopenhyan and Rogerson (1993).

Table 3 shows the calibration results. It can be seen that the size distribution of firms, the share of vertically integrated firms, and the employment to population

[14] Hall (1993), De Loecker et al (2009), Corchon and Moreno (2010), and Moreno and Rodriguez (2010) estimate a markup of 16%.

ratio are well fitted. Figure 3 documents the firm size distribution in the model and in the US data.

Table 3: Data moments and model moments.

	Data	Model
Size distribution of firms*	%	%
1-9	78.61	82.16
10-19	10.68	11.13
20-99	8.88	4.55
100-499	1.52	1.76
500-999	0.15	0.27
1000-4999	0.13	0.10
5000+	0.03	0.03
Share of vertically integrated firms	8-9	8.62
Employment to population ratio	60	63

* Source: Census Bureau 2008

Figure 3. Density of firm sizes.

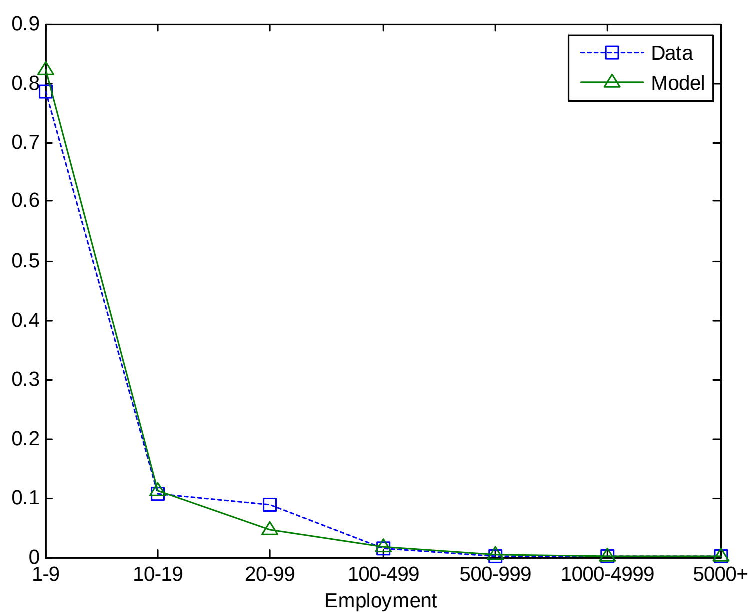


Figure 4 documents the equilibrium size distribution of firms together with the size distribution of vertically integrated and not vertically integrated firms. The line with squares represents the total size distribution of firms, while the other two lines represent, for each size, the proportion of each type of firm (vertically integrated and not vertically integrated) to the total share of firms for each particular size (this is, the area below each line adds up to the share of each category in the total number of firms). The line with circles represents the size distribution of entrants.

Figure 4. Density of firm sizes.

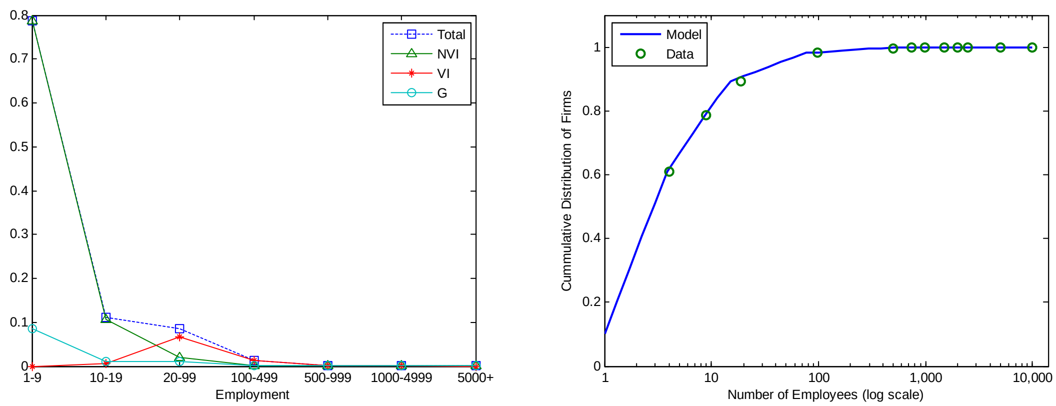
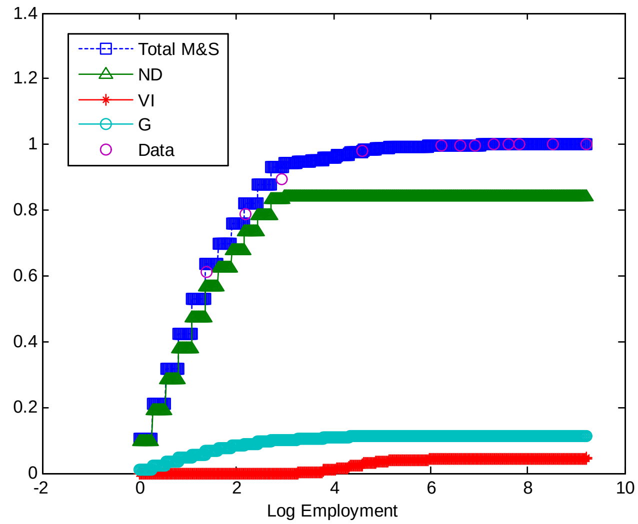


Figure 5 presents the cumulative fraction of firms. The line with squares represents the total size distribution of firms in the model and the circles represent the one corresponding to the data. The lines with triangles and stars correspond to the cumulative distribution of not vertically integrated and vertically integrated incumbent firms in equilibrium. Notice that the size distribution of vertically integrated firms dominates (in first order stochastic dominance sense) to the size distribution of not vertically integrated ones as in Hortaçsu and Syverson (2009).

Figure 5. Size distribution of all firms, vertically integrated and not vertically integrated firms.



3.3.2 The effect of size-dependent policies

The model economy presented above, gives rise to rich firm behavior as manufacturer enter, exit, and decide how to obtain their inputs. In this environment an industrial structure emerges as the result of optimal investment decisions. In this framework, regulations that restrict the size of establishments affect firms' incentives to use their optimal vertical structure and have an impact on profitability, size distribution of firms and average productivity. In this section we use the model to analyze how size-dependent policies affect TFP by changes in the optimal vertical structure of firms.

I introduce positive idiosyncratic distortions to employment and production, τ_n and τ_y respectively. In order to do so, in the benchmark economy (without distortions) I first find the idiosyncratic productivity level for which the firm size is the mean, $\widehat{z}$, and then tax all firms with idiosyncratic productivity levels above $\widehat{z}$. Table 4 documents the results for distortions in employment, τ_n, of 5%, 10%, and 15% and compares the results with the first column that correspond to the benchmark economy.

Focusing on the economics of the model, the results indicate that distortions on employment act as a subsidy to low productivity firms and as a barrier to vertical integration (penalizing high productivity firms). Regarding the manufacturers with high productivity, for the ones that prefer to use specialized inputs, vertical integration becomes less attractive and they increase the demand for standardized inputs. For manufacturers with high productivity that prefer to use standardized inputs, the tax on employment gives incentives to reduce their size. For the mid-sized firms, since no distortions are imposed to them, as it is explained later on, both the increase in the final good price as well as the decrease in the standardized input price (that resembles a subsidy), induce them to expand. The same happens to firms that are smaller than the mid-sized firms.

Formally, the distortion on employment reduces manufacturers expected value at entry. In order to restore the free entry condition, the final good price must increase. Since the mean demand for standardized inputs ($\overline{m}$) increases supplier's expected value at entry increases (upon entry suppliers sell more inputs increasing profitability). In order to restore the free entry condition for suppliers the standardized input price must decrease. As a result there are two effects on the mean firm size. First, since P increases and P_s declines, and as no distortions are imposed to small firms, small firms expand. Second, as explained before, big firms contract and the fraction of vertically integrated firms declines. As the second effect dominates the mean size of firms decreases.

Through this mechanism, distortions on employment have a nontrivial effect on the size distribution of firms. Figure 6 shows that the density of mid-sized firms increases. There is a reallocation of resources (employment) from big firms to small firms.

Table 5 shows the size distribution of firms by vertical structure. It can be noticed that there is a decline in the fraction of mid-sized firms that vertically integrate. Furthermore, there is a shift to the right of the size distribution of not vertically integrated firms, while the size distribution of vertically integrated firms shifts to the left.

There are two effects on TFP. First, employment is shifted from high to low productivity firms. Second, distortions on employment strongly penalize to vertically integrated firms (the most efficient firms), thus manufacturers that continue operating under vertical integration reduce their size, and fewer manufacturers become vertically integrated. As a result, there is a decline in total factor productivity.[15]

[15] See the appendix for definitions of total factor productivity.

Table 4: Comparative Statics on τ_n.

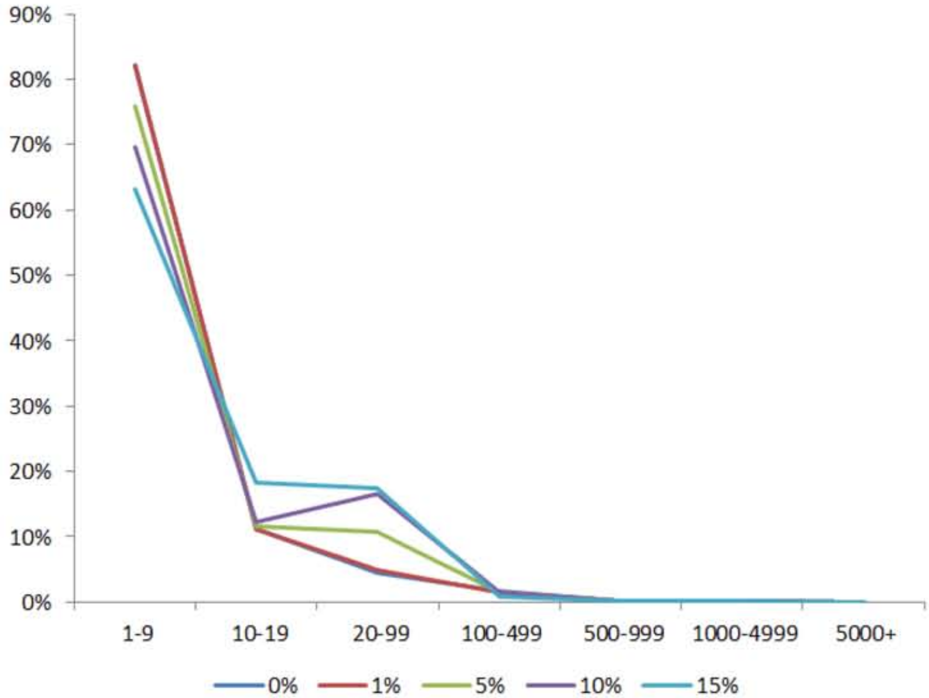
Tax	τ_n 0%	5%	10%	15%		τ_n 0%	5%	10%	15%
Size dist. of firms									
1-9	82.16	75.85	69.55	63.26	P	1.00	1.03	1.06	1.09
10-19	11.13	11.69	12.21	18.26	$\overline{m}$	6.08	7.88	9.79	11.67
20-99	4.55	10.72	16.55	17.47	P_s	1.16	1.12	1.10	1.08
100-499	1.76	1.45	1.42	0.85	Mean size	100.0	98.6	98.8	99.0
500-999	0.27	0.16	0.18	0.06	NVI	100.0	129.4	160.8	191.8
1000-4999	0.10	0.10	0.08	0.08	VI	100.0	94.4	90.3	86.5
5000+	0.03	0.03	0.02	0.02	TFP	100.0	98.7	98.0	97.6
Exit rate	10	10	10	10					
% VI firms	8.6	8.2	7.7	7.1					

Note: $\overline{m}$ is the mean standardized inputs demand.

Table 5: Comparative Statics on τ_n.

Tax	0%		5%		10%		15%	
Size distribution of firms	NVI	VI	NVI	VI	NVI	VI	NVI	VI
1-9	86	0	79	0	72	0	66	0
10-19	12	0	12	0	13	0	19	0
20-99	2	57	8	64	15	65	15	77
100-499	0	35	0	30	0	29	0	19
500-999	0	6	0	4	0	3	0	1
1000-4999	0	2	0	2	0	2	0	2
5000+	0	1	0	1	0	1	0	1

Figure 6. Size distribution of all firms and τ_n.



The introduction of distortions in production, τ_y, is very important because it helps to understand further how the model works. It shows the new insights the mechanism in the current paper provides. Furthermore, it shows a key result that highlights how an economy reacts to different sources of distortions.

Table 6 documents the results for distortions in production, τ_y, of 5%, 10%, and 15%, related to the benchmark economy. By comparing the results with the previous exercise, it can be noticed that distortions on production have a bigger impact on prices P and P_s than distortions on employment.[16]. Formally, the elasticity of the expected value at entry for both manufacturers and suppliers with respect to distortions in production, τ_y, is higher than with respect to distortions on employment, τ_n. Through the effects on price, as it is explained later on, distortions in production have the potential to do much more damage (the misallocation is higher) as it is reflected by an increase in the mean size of firms and a bigger decline in TFP.

Formally, under the same reasoning as before, as τ_y increases the expected value at entry decreases. To restore the free entry condition of manufacturers the price of the final good increases (see Table 6). As vertical integration decreases and firms demand more standardized inputs, the mean demand for inputs increases and the input price decreases (so as to restore the free entry condition for suppliers). As a result there are two effects on the mean firm size. First, as no distortions are

[16] The magnitude of the subsidy mid-sized firms receive (by means of prices) is bigger.

imposed to small firms, the increase in final good price and the decline in the input price induce small firms to expand. Second, big firms contract and the fraction of vertically integrated firms declines. In contrast with the case of distortions on employment, for the case of distortions on production the first effect dominates and thus the mean size of firm increases.

Regarding the effects of τ_y on the size distribution of firms Figure 7 shows a bigger increase in the density of mid-sized firms compared to the case when τ_n increases. There is a reallocation of resources (employment) from big firms to small firms. Table 7 indicates that firms that do not become vertically integrated increase their size, while vertically integrated firms reduce their size (there is a shift to the right of the size distribution of not vertically integrated firms, while the size distribution of vertically integrated firms change in the opposite direction). The fraction vertical integration among mid-sized firm decreases. Firms with low productivity expand more with τ_y than they do with τ_n, and thus the misallocation is more important. In contrast with the case in which τ_n is positive, there is an increase in the mean size of firms and thus a bigger decline in total factor productivity.

Table 6: Comparative Statics on τ_y.

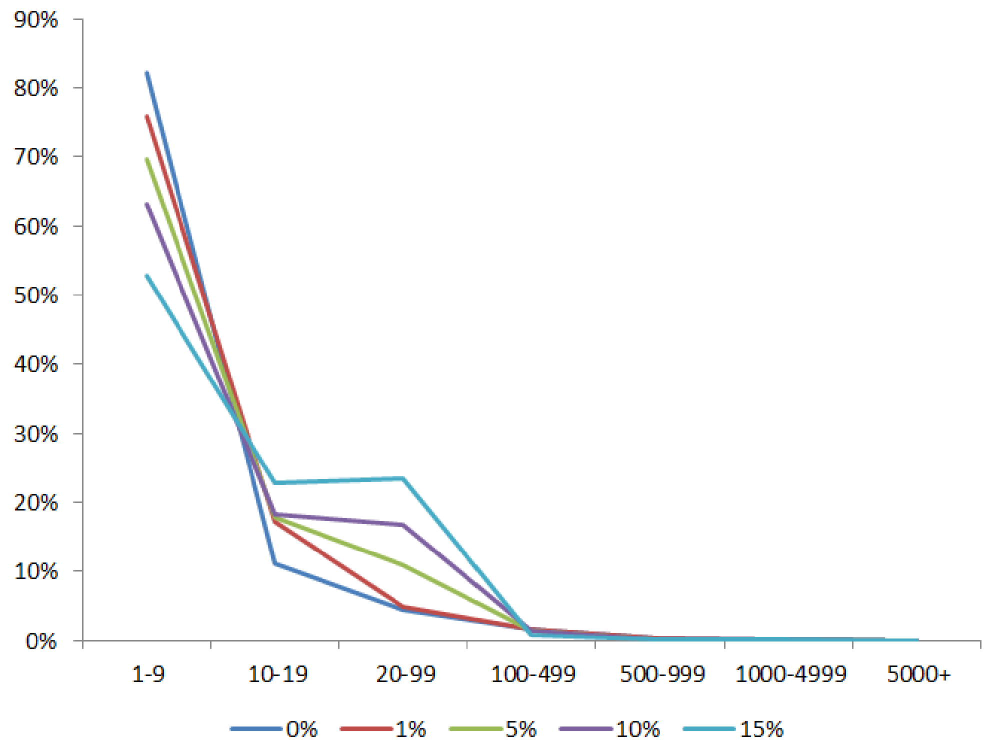
	τ_y					τ_y			
Tax	0%	5%	10%	15%		0%	5%	10%	15%
Size dist. of firms									
1-9	82.16	69.62	632.3	52.75	P	1.00	1.04	1.08	1.12
10-19	11.13	17.78	18.25	22.73	Mean $\overline{m}$	6.08	8.65	11.57	14.21
20-99	4.55	10.87	16.70	23.50	P_s	1.16	1.11	1.08	1.07
100-499	1.76	1.45	1.44	0.86	Mean size	100.0	103.2	106.9	110.7
500-999	0.27	0.16	0.18	0.06	NVI	100.0	141.9	190.0	233.5
1000-4999	0.10	0.10	0.08	0.08	VI	100.0	96.0	92.0	88.1
5000+	0.03	0.03	0.02	0.02	TFP	100.0	98.3	97.4	97.1
Exit rate	10	10	10	10					
% VI firms	8.6	7.9	7.1	7.0					

Note: $\overline{m}$ is the mean standardized inputs demand.

Table 7: **Comparative Statics on τ_y.**

Tax	0%		5%		10%		15%	
Size distribution of firms	NVI	VI	NVI	VI	NVI	VI	NVI	VI
1-9	86	0	73	0	66	0	55	0
10-19	12	0	19	0	19	0	24	0
20-99	2	57	9	62	15	68	21	78
100-499	0	35	0	31	0	25	0	19
500-999	0	6	0	3	0	5	0	1
1000-4999	0	2	0	2	0	2	0	2
5000+	0	1	0	1	0	1	0	1

Figure 7. Size distribution of all firms and τ_y.



3.4 Conclusion

In the current paper I have developed a dynamic model of an industry with heterogeneous firms interacting as buyers and sellers of inputs, endogenous vertical integration, and market frictions to study how size-dependent policies interact with the organization of production that firms optimally choose determining differences in the size distribution of firms and TFP across countries. In this context, distortions on production and employment generates a reallocation of resources (employment) from big firms to small firms and act as barriers to vertical integration. I find that a 15% output tax on firms that are above mean level of productivity generates a decline in the fraction of vertically integrated firms from 8.7% to 7%, a decrease in TFP of 2.9%, and an increase of 11% in the mean size of firms. A 15% tax on employment generates a decline in the fraction of vertically integrated firms from 8.6% to 7.1%, a decrease in TFP of 2.4% and a decline of 1.2% in the mean size of firms.

3.5 Appendix

3.5.1 Solution Method

The algorithm to compute the equilibrium is as follows:

1) Take initial guesses for the price of the final good, p^0, for the input price, p_s^0, and for the supplier's acquisition price P^{VI0}.

2) Take an initial guess for the density of productivity of manufacturers looking for a standardized suppliers $J_0(z,\tau)$,

3) Obtain policy functions $n(z,\tau,)$, $m(z,\tau)$, $x'(z,\tau,\varepsilon)$, $n_{VI}(z,\tau,\varepsilon)$, $m_{VI}(z,\tau,\varepsilon)$ and value functions , $V(z,\tau,\varepsilon)$, $V^{VI}(z,\tau,\varepsilon)$, $W(z,\tau,\varepsilon)$.

4) Compute the price for $P^{VI0}(z,\varepsilon) = \sum\limits_{\varepsilon}\sum\limits_{z}\sum\limits_{\tau} W(z,\tau,\varepsilon;p,p_s)J(z,\tau)g^s(\varepsilon)$.

5) Use the computed decision rules to compute the invariant density of productivity of manufacturers looking for a standardized suppliers $J(z,\tau)$, and compare it with $J_0(z,\tau)$:

   i) If they are not close$\Rightarrow$guess a new one ($J_0(z,\tau) = J(z,\tau)$) and repeat from point (3) until they get close.

   ii) If they are close$\Rightarrow$stop and go to next point .

6) Compute $V_e(p,p_s)$ and $W_e(p,p_s)$ and given the entry costs C_e^d and C_e^s verify if free entry conditions (equations 7 and 8) hold:

   i) If they do not hold:

      * If $V_e(p,p_s) < C_e^d$ and/or $W_e(p,p_s) < C_e^s \Rightarrow$ guess a new higher prices, p and p_s by bisection and repeat from point (1).
      * If $V_e(p,p_s) > C_e^d$ and/or $W_e(p,p_s) > C_e^s \Rightarrow$ guess a new lower prices, p and p_s by bisection and repeat from point (1).

   ii) If $V_e(p,p_s) \approx C_e$ and $W_e(p,p_s) \approx C_e^s$ $\Rightarrow$stop and go to next point.

7) Use the computed decision rules to compute the fixed points of the distribution of manufacturers when the mass of firms is one ($M^d = 1$). Thus, we have the fixed points $\widehat{\Phi}$ and $\widehat{\Phi}^{VI}$.

8) Compute the aggregate profits and tax revenue, $\Pi(p,p_s,w,1,1)$ and $R(p,p_s,1,1)$ and obtain aggregate labor demand and supply. Use the linear homogeneity of labor demand and supply to obtain the equilibrium value for M^d (given $M^d = M^s$) that satisfies the labor market clearing condition for the final good: $L^d(p,p_s,w,M^d,M^s) = L^s(p,p_s,w,\Pi(p,p_s,w,M^d,M^s) + R(p,p_s,w,M^d,M^s))$.

3.5.2 Total factor productivity

In this section I describe how the total factor productivity is calculated. I define TFP as the revenue total factor productivity. It is calculated as follows:

$$TFP = \sum_{z}\sum_{\tau}\sum_{\varepsilon}\frac{pzn(z,\tau)^{\alpha}}{(w+p_s)n(z,\tau)}\widetilde{\Phi}(z,\tau,\varepsilon)+\sum_{z}\sum_{\tau}\sum_{\varepsilon}\frac{pzn_{VI}(z,\tau,\varepsilon)^{\alpha}}{(w-c(z,\varepsilon))n_{VI}(z,\tau,\varepsilon)+C_f^{VI}}\widetilde{\Phi}^{VI}(z,\tau,\varepsilon)$$

$$+\sum_{z}\sum_{\tau}\sum_{\varepsilon}\frac{p_s n(z,\tau)}{wn(z,\tau)}\widetilde{\Xi}(z,\tau,\varepsilon)$$

where the first term represents the weighted average (the weight is the share of not vertically integrated manufacturers in each state, $\widetilde{\Phi}(z,\tau,\varepsilon)$) of the ratio of not vertically integrated manufacturer's revenues, $pzn(z,\tau)^{\alpha}$, to their total production cost, $(w+p_s)\ n(z,\tau)+\ C_f$. The second term is the weighted average of the ratio of vertically integrated manufacturer's revenues to their corresponding total production costs. In this case $\widetilde{\Phi}^{VI}(z,\tau,\varepsilon)$ is the share of vertically integrated manufacturers in each state (z,τ,ε). In contrast with the first term, in the denominator it appears the variable cost advantage of specific investments, $c(z,\varepsilon)$ and fixed cost of a vertically integrated firm, C_f^{VI}. The last term correspond to the revenue TFP of suppliers. The function $\widetilde{\Xi}(z,\tau,\varepsilon)$ is the share of suppliers matched to a manufacturer of type (z,τ,ε).

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